UNIVERZITA MATEJA BELA V BANSKEJ BYSTRICI
FAKULTA PRÍRODNÝCH VIED

VYUŽITIE METÓDY CLIL (CONTENT AND LANGUAGE INTEGRATED LEARNING) VO VÝUČBE TEMATICKÉHO OKRUHU KOMBINATORIKA, PRAVDEPODOBNOSŤ A ŠTATISTIKA V GYMNÁZIÁCH

Diplomová práca

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Pracovisko:	Katedra matematiky, Fakulta prírodných vied, Univerzita Mateja Bela, Banská Bystrica
Vedúci diplomovej práce:	Doc. RNDr. Miroslav Haviar, CSc.

Banská Bystrica 2013 **Bc. Alžbeta Brišová**

Čestné vyhlásenie

„Vyhlasujem, že som diplomovú prácu vypracovala samostatne, pod odborným vedením vedúceho diplomovej práce. Použila som literatúru uvedenú v referenciách.“

V Banskej Bystrici, 19.04.2013

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Poďakovanie

Ďakujem vedúcemu mojej diplomovej práce doc. RNDr. Miroslavovi Haviarovi, CSc. a Mgr. Daniele Guffovej za ich pomoc, ochotu, trpezlivosť a odborné rady a pripomienky, ktoré mi veľmi pomohli pri vypracovávaní diplomovej práce.

Abstrakt

Brišová, Alžbeta: *Využitie metódy CLIL (Content and Language Integrated Learning) vo výučbe tematického okruhu Kombinatorika, pravdepodobnosť a štatistika v gymnáziách.* [Diplomová práca]. Univerzita Mateja Bela v Banskej Bystrici. Fakulta prírodných vied, Katedra matematiky. Vedúci práce: Doc. RNDr. Miroslav Haviar, CSc. Stupeň odbornej kvalifikácie: Magister – Mgr. Banská Bystrica, 2013. 91 s.

Hlavným cieľom diplomovej práce bolo navrhnúť a otestovať ukážkové hodiny v rámci tematického okruhu Kombinatorika, pravdepodobnosť a štatistika v gymnáziách s využitím metódy CLIL. Práca je rozdelená na teoretickú a praktickú časť.

Teoretická časť obsahuje šesť kapitol. V prvých troch sú ozrejmené základné pojmy súvisiace s metódou CLIL (Content and Language Integrated Learning) v integrovanej výučbe matematiky a anglického jazyka. Štvrtá kapitola uvádza výhody opisovanej metódy. V posledných dvoch kapitolách je stručne charakterizovaná aplikácia metódy v edukácii na území Slovenskej republiky.

Praktická časť je rozdelená na päť častí. Prvá predostiera prípravy ukážkových hodín. Jednotlivé prípravy, pracovné listy a prezentácie sú predmetom druhej kapitoly. Tretia kapitola charakterizuje študentov a študentky, ktorí sa zúčastnili výučby využívajúcej metódu CLIL. V štvrtej kapitole je opísaná miera znalostí, nadobudnutých prostredníctvom integrovanej výučby matematiky a anglického jazyka. Znalosti boli zistené metódou pozorovania a didaktickým testom. V záverečnej kapitole praktickej časti je opísaný postoj študentov k realizovanej výučbe zistený metódou dotazníka.

Práca je napísaná v anglickom jazyku.

Kľúčové slová: matematika, anglický jazyk, metóda CLIL, vyučovanie matematiky, integrácia matematiky a anglického jazyka, kombinatorika, pravdepodobnosť, štatistika.

Abstract

BRIŠOVÁ, Alžbeta: *Using CLIL Method (Content and Language Integrated Learning) in Teaching and Learning the Combinatorics, Probability and Statistics Theme at Grammar Schools.* [Diploma thesis]. Matej Bel University, Banská Bystrica. Faculty of Natural Sciences, Department of Mathematics. Supervisor: Doc. RNDr. Miroslav Haviar, CSc. Qualification degree: Master degree – Mgr. Banská Bystrica, 2013, 91 p.

The main aim of the thesis was to design and verify materials which use CLIL method as a tool in teaching and learning the Combinatorics, Probability and Statistics theme at grammar schools. The thesis is divided into theoretical and practical parts.

Theoretical part consists of six chapters. The first three chapters present the concepts related to the CLIL method (Content and Language Integrated Learning) in the integration of Mathematics and English language. The fourth chapter provides an overview of the advantages of the used method. The last two chapters briefly characterize the application of this method in the education in the Slovak Republic.

The practical part is divided into five chapters. The first chapter describes lesson planning. Particular lesson plans, worksheets and presentations are present in the second chapter. The third chapter characterizes learners who participated in the learning and teaching via CLIL method. The fourth chapter ascertains the level of knowledge which students acquired through the integration of Mathematics and English language. It was determined via observation and didactic test. The last chapter of this part focuses on the attitude of learners to teaching and learning via CLIL method. Their attitude was ascertained through the method of questionnaire.

The thesis is written in English language.

Key words: Mathematics, English language, CLIL method, Mathematics education, integration of Mathematics and English language, Combinatorics, Probability, Statistics.

MATEJ BEL UNIVERSITY IN BANSKÁ BYSTRICA
FACULTY OF NATURAL SCIENCES

USING CLIL METHOD (CONTENT AND LANGUAGE INTEGRATED LEARNING) IN TEACHING AND LEARNING THE COMBINATORICS, PROBABILITY AND STATISTICS THEME AT GRAMMAR SCHOOLS

Diploma Thesis

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Study Programme:	Teacher Training in Mathematics and Teacher Training in English language and literature (Teacher Training, Master II. degree, full-time study)
Field of Study:	1.1.1. – Teacher Training in Academic Subjects
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Banská Bystrica 2013 **Bc. Alžbeta Brišová**

PREFACE

This diploma thesis covers the implementation of the CLIL (Content and Language Integrated Learning) method in the integration of Mathematics and English Language. It provides an overview of the concepts related to this integration, its advantages and contributions to the learning and teaching. This approach becomes widely employed across the Europe because it suits the demands on the present-day education. The integration of Mathematics and English Language with its benefits and challenges for learners and teachers in the European context and also the impact of this innovative method on the holistic personality development of students were the main reasons for elaborating this theme.

In the Slovak Republic, the provision of the CLIL method faces the lack of materials for teachers. Designing and testing of the lesson plans and their related worksheets or power-point presentations which use CLIL method as a tool in teaching and learning the Combinatorics, Probability and Statistics theme at grammar schools are the main contributions of this thesis.

The initiatives aimed at ascertainment of the knowledge which learners gained throughout the lessons. Abiding interest was in attitude of learners towards this type of educational provision.

Materials included in this thesis can be used by teachers of Mathematics who want to use the CLIL method to enhance education of learners at grammar schools.

CONTENT

THE LIST OF PICTURES, GRAPHS AND TABLES

INTRODUCTION

'*Live as if you were to die tomorrow.*
Learn as if you were to live forever.'
Mahatma Gandhi

Multilingualism is becoming an important part of the European culture. Education authorities have recognized the value of the CLIL (Content and Language Integrated Learning) method in helping European citizens to cope with requirements of the world around them. Consequently, the CLIL approach is becoming increasingly widespread across the Europe since it not only satisfies demands on mainstream education but also meets core standards within the European context.

This approach features prominently in the integration of the non-language subject and the foreign, regional or minority language. It seeks the proficiency in both and attaches the same importance to each.

The Slovak Republic has had long-term experience with the CLIL method. The European programmes in education and training has served as a catalyst for further spread of the employment of the CLIL approach across the country. However, the use of this method has been challenged by many difficulties concerning shortage of trained teachers, lack of material suitable for CLIL provision and others.

This diploma thesis deals with the CLIL method in the integration of Mathematics and English language. The focus is on CLIL method as a tool in teaching and learning the Combinatorics, Probability and Statistics theme at grammar schools.

The main reasons for elaborating this theme are its advantages and challenges for learners and teachers in the integration of Mathematics and English language. Other motive is our interest in innovative methods which have impact on the holistic personality development of learners. CLIL is undoubtedly ranged between those methods.

The thesis is divided into theoretical and practical part. The theoretical part consists of six chapters. The first three chapters present the concepts related to the CLIL method (Content and Language Integrated Learning) in the integration of Mathematics and English Language. The fourth chapter provides an overview of the advantages of this method. The last two chapters acquaint readers with the status of CLIL in Slovakia and its correspondence with the national educational policy.

The practical part is divided into five chapters. The first two chapters cover materials which encompass teaching and learning the CPS theme via CLIL method at grammar schools. Designing and verifying of the materials can be considered as the main contribution of this thesis.

The third chapter provides the profile of learners from two different grammar schools, who participated in the learning and teaching via CLIL method.

The fourth chapter deals with the ascertainment of the level of knowledge which students acquired through the integration of Mathematics and English language. It was ascertained via observation and didactic test.

The last chapter of this part provides a closer look at the attitude of learners towards this type of educational provision; to what extent they acknowledged CLIL advantages and challenges. We took an interest if they consequently seized the opportunity to enhance their own education. To find out this information, learners were asked to fill in the questionnaires.

The implementation of the CLIL method still faces many challenges; one of them is the lack of material appropriate for the education via CLIL. Materials included in this thesis can enable teachers of Mathematics to use the CLIL method to enhance education of learners at grammar schools.

1 THEORETICAL PART

1.1 CLIL as a Teaching Method

CLIL is an acronym for Content and Language Integrated Learning. This term was introduced by David Marsh, University of Jyväskylä, Finland (1994): *'CLIL refers to situations where subjects, or parts of subjects, are taught through a foreign language with dual-focused aims, namely the learning of content and the simultaneous learning of a foreign language.*'

The characteristic feature of CLIL is integration. In parallel with developing of the non-language content of a curriculum subject through the medium of the target language, the target language is developed via the non-language content. It is a method of far broader scope than language teaching and learning (Baïdak et al., 2006).

This dual-focused educational approach, which challenges the character of traditional education by its humanizing character, is now widely employed across Europe (Baïdak et al., 2006).

Although the CLIL definition has been coined in 1994, the idea of integrating the content and language has been implemented around the world for centuries – from the Akkadian period (around 3000 BC) to the introduction of bilingual education in the early sixties (Marsh et al., 2008). So the CLIL approach as such is both old and new.

The new about CLIL is that it synthesizes and applies the knowledge from various educational approaches, such as immersion, multilingual education, language showers[1] and others (Marsh et al., 2008). Moreover, one of the core principles is that CLIL contributes to the quality of teaching and learning which all students are entitled to have.

1.2 Aims of CLIL

In accordance with *The conceptual framework of teaching foreign languages at primary and secondary schools* (*Koncepcia vyučovania cudzích jazykov v základných a stredných školách*), CLIL aims at preparing students for a world that relies more and more on multilingual exchanges. The target language becomes a tool for cognitive

[1] 'Language showers include games, songs and others in the target language. This developing of routines creates sense of security, lowers anxiety and boosts learning.' (Marsh et al., 2008, p. 13)

and intercultural growth. The idea of CLIL is to offer linguistically-enhanced education with the result of 'functional bilingualism' of learners. It means that a student has the knowledge of two languages for a certain area of a curriculum subject, in our case it is Mathematics.

Referred to *CLIL Handbook for teachers* (2008), the introduction of new ideas and concepts in curriculum subject is the main focus of CLIL. Apart from that it aims to:

- improve the performance of learners in the curriculum subject and the target language;
- increase motivation to study foreign languages as well as the curriculum subject;
- encourage stronger links with the citizenship curriculum;
- increase confidence of learners in the target language;
- provide cognitively challenging materials from the beginning;
- provide scaffolding[2] to support learning of content and language.

1.3 Principles of CLIL

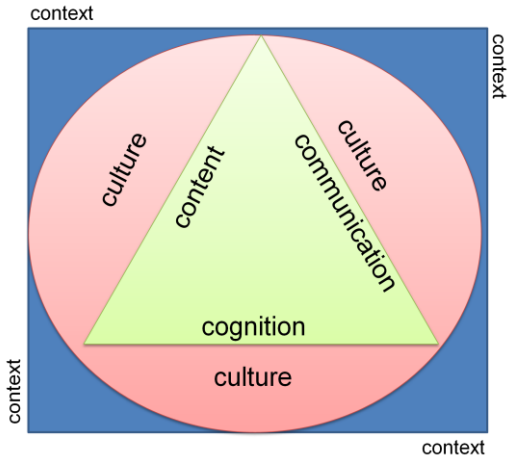
In order to take account of a wide range of contexts, the CLIL approach is flexible. However, there are some fundamental principles which are recognized as essential to make CLIL effective and to distinguish it from other kinds of learning and teaching in non-native language. In *Content and Language Integrated Learning* (Coyle et al., 2010) is asserted that in its most reduced state, the following guiding principles can be said to drive this model:

1) **content –** progression in new knowledge, skills and understanding relating to the successful content learning lies at the heart of the learning process;
2) **communication** – permanent interaction and improvement in language using and learning;
3) **cognition** – CLIL challenges learners to create new knowledge and helps them to develop thinking skills and understanding through engagement in cognitive processes and reflection;

[2] '*Scaffolding content and language* – breaking down tasks into small steps, creating interest, providing constructive feedback, use of language frames, substitution tables, word banks, glossaries, use of native language, use of models for production of language.' (http://www.cambridgeenglish.org/images/22191-tkt-clil-handbook.pdf)

4) **culture** – progression towards pluricultural understanding, 'self' and 'other' awareness, identity, community and citizenship.

Although these principles, known as 4 Cs, can be outlined individually, their connection into an integrated whole is fundamental in this method (Coyle et al., 2010). For better understanding of CLIL method, see the following conceptual diagram which offers holistic and symbiotic view.



Picture 1: **The conceptual diagram of CLIL**[3]

We have introduced principles of CLIL approach to learning and teaching. Since the most challenging principle to be applied is the communication principle, we deal with its implementation in the following sections.

1.3.1 Language Triptych

The central focus of CLIL method is on linguistic progression – language learning and language using. When transforming this theory into practice, a teacher should be aware of three interrelated types of languages used for different purposes (Coyle et al., 2010):

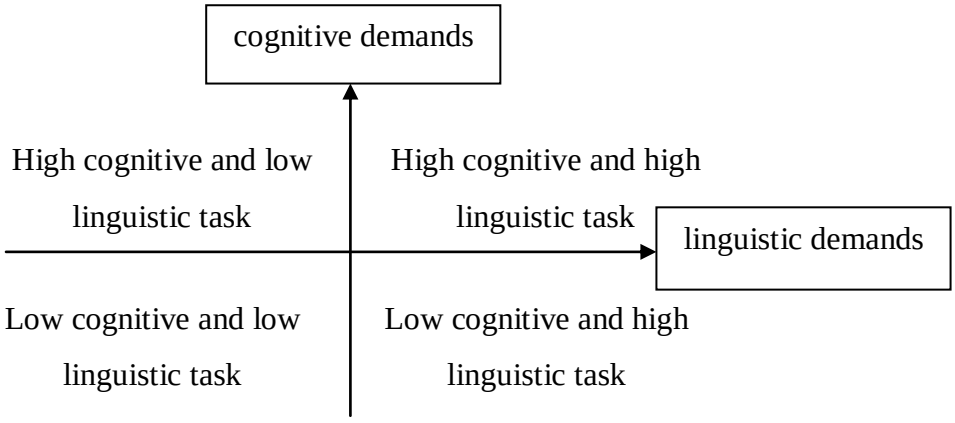
- **language *of* learning** – the content obligatory language the learners need to access new knowledge and understanding (not only key words but also the way they are used, vocabulary, phrases, grammatical demands,...); e.g. when dealing with the topic *Probability*, learners need to operate with words such as the *probability of (doing), be equally likely* and they need to use *modal verbs to predict future* and *language of describing, defining, explaining, hypothesizing;*

[3] Source: http://www.slideshare.net/ydgs20/session-6-13820161.

- **language *for* learning** – the most crucial element for successful CLIL, the language needed to operate in foreign language environment (for pair/group work, asking questions, debating, etc.); e.g. when dealing with probability, learners asks questions: *What do you think is more probable...?* or *What are possible outcomes when ...?;*
- **language *through* learning** – new language that cannot be planned. This emerging language needs to be captured, recycled and developed so that it becomes a part of a repertoire of learners. When learning topic *Probability* we expect students to know some of the uses of the present simple tense. During the lesson, students come across with its other uses – e.g. when describing general truths, facts and scientific laws.

1.3.2 Auditing Tasks

Tasks introduced during the lesson can differ in their linguistic or cognitive demands. So we distinguish four types of tasks as it is shown in the following picture adapted to *CLIL Content and Language Integrated Learning* (Coyle et al., 2010):



Picture 2: **Types of tasks**

The logic order of tasks throughout the lesson is usually from linguistically accessible and cognitively non-demanding to high cognitive and high linguistic demanding. The target type of task is the latter mentioned. The role of a teacher is to modify tasks to be compatible with the national standards and make them linguistically accessible for students (Coyle et al.,2010).

1.4 Advantages of CLIL

CLIL is of unusual interest for its promotion of language learning and teaching and language diversity. Students are given an opportunity to acquire proficiency in languages. Having taken this opportunity, they are better prepared for mobility in Europe which is becoming more widespread and inevitable.

Students who participate in CLIL lessons enjoy considerable advantages of CLIL, as follows:

- focus on *pragmatic knowledge* and skills and the promotion of *linguistic competency* and spontaneity;
- development of conceptual skills of learners, literally *the way how to think;*
- *the quality of learning* - it enriches understanding of concepts which results in better associations of concepts and leads to more sophisticated way of learning;
- development of methods that are very specific to the dual-focused aims leads to better *quality of teaching*;
- through *interpersonal* and *intercultural communication* CLIL provides students opportunities for learning languages;
- it satisfies the need of learners to be exposed to a situation calling for *genuine communication;*
- *naturalness* of the environment via bringing real-life situations into the classroom;
- *inclusiveness* – CLIL is appropriate for all learners regardless their age or linguistic level;
- curriculum linking: new *challenges* (Coyle et al., 2010, Marsh – Mehisto, 2008).

Above mentioned benefits and other merits of CLIL exceed general demands on mainstream education. Other benefits are worth mentioning – its diversity in forms of implementation, target audience and adaptability to local conditions which also contributed to the wide employment of this approach across the Europe.

Schools implementing this type of tuition achieve to *increase the number of lessons* for foreign languages without increasing the number of lessons allotted to the foreign languages as curriculum subjects.

1.5 CLIL in the Slovak Republic

Slovakia has had long-term experience with CLIL method, including various languages, and schools spread across the country. Bilingual education has been provided

since the early 1950s in minority languages and since 1990s in foreign languages. In 2003 legislation related to specific aspects of bilingual education has been developed and laws have reflected the experience of the bilingual schools (Lauková, 2005). Slovakia, as a part of the European Union has implemented recommendations of the European Council (2005 onwards) that CLIL should be adapted as a major educational initiative.

Based on *The Current Report: Didactic Efficiency of the CLIL Method at the First Level of Primary Schools in the Foreign Languages Teaching (Priebežná správa: Didaktická efektívnosť metódy CLIL na prvom stupni ZŠ vo vyučovaní cudzích jazykov)* CLIL is available at primary and secondary levels of education. Since 2008 to 2012 there was a pilot project on CLIL which included 14 primary schools.

Status of languages taught in CLIL (Baïdak et al., 2006) in the Slovak Republic:

- foreign languages: English, French, German, Spanish and Russian;
- regional and/or minority languages: Hungarian, Ukrainian and Ruthenian.

In response to the current EU education policy, there is a growing demand to introduce CLIL to vocational and business schools. Synthesis of the professional and language skills is crucial for the graduates to become successful in their occupations and contributes to the mobility which should be within the reach of everyone (Lauková, 2007).

However, the shortage of training and recruitment of appropriate teachers is one of the major barriers to implement this method. Slovak teachers are not formally trained in CLIL approach and they are insufficiently qualified or competent. They can participate in short-term courses usually provided by foreign training institutions. To set-up in-service education is one of the issues concerning its future development (Kubeš, 2011).

Another current issue is the lack of teaching and learning materials for CLIL lessons (worksheets, lesson plans and teaching instructions) based on *The Slovak Curriculum* and adapted directly for Slovak teachers (Kubeš, 2011). Cooperation among schools in order to exchange and share experiences with CLIL is inevitable step for efficient implementation of this type of tuition at Slovak schools (Lauková, 2007).

1.6 The Slovak Curriculum

In the Slovak Republic *The Slovak Curriculum (Štátny vzdelávací program[4])* is the supreme curricular document which contains the general objectives and requirements related to the content of education and training and the key competencies. It states the education standards but leaves room for adding content according to regional or other requirements of schools. The specificities of the school imply the school orientation and suggest the school profile.

The Slovak Curriculum defines the requirements and objectives related to the subject Mathematics within the context of mathematical literacy and competencies. In case of English language it is language literacy and competencies. The supreme curriculum recommends using extra-curriculum activities and topics.

1.6.1 Mathematical Literacy and Competencies

Mathematical literacy is 'a capacity of an individual to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that life of an individual as a constructive, concerned and reflective citizen' (PISA, 2009, p.14).

The assessment framework for mathematical literacy makes reference to situations (personal, educational, occupational, public and scientific), mathematical content and mathematical processes (PISA, 2009).

One of the content areas in Mathematics is Combinatorics, Probability and Statistics which is the central theme of this thesis. Pursuant to *The Slovak Curriculum, Mathematics, Appendix ISCED 3A (ŠVP, Matematika – príloha ISCED 3A*), students should improve in wide range of mathematical processing, such as logical reasoning and argumentation, problem solving, data representation, situation modelling, applying symbolic elements and communication. The level of each competency can differ, from the basic one – the reproduction through the connection to the reflection.

To apply mathematical literacy in everyday problems, it is important to gain key competencies which facilitate functional use of the knowledge and skills.

[4] Štátny vzdelávací program, abbr. ŠVP.

1.6.2 Language Literacy and Competencies

Language education policy in Slovakia acts in accordance with the *Common European Framework of Reference for Languages*[5]. It is helpful to define desired language proficiency levels at different levels of educational system.

Language literacy encompasses the ability to function communicatively in everyday situations. According to the level of efficiency of the learner in the language, CEFR divides users of the language into three broad divisions. Each of them can be divided into two levels. Pursuant to *The Slovak Curriculum, Appendix ISCED 3*, students of upper secondary level of education should be *independent users* of the language (the level corresponds to the B2 level in the CEFR).

According to CEFR, *independent users* can understand the main ideas of text about factual as well as about abstract topics, including technical discussions in their field of specialisation. Another required competency for *independent users* is their unstrained interaction with other speakers of the language with a degree of fluency and spontaneity. Learners are able to produce detailed text on a wide range of subjects (based on CEFR).

Having mentioned the advantages of CLIL method, this tuition can contribute to the considerable extent to the acquisition of these competencies (Marsh et al., 2008). Throughout the tuition, learners are encouraged to explain a viewpoint on topical issues, reasoning their advantages and disadvantages which make contribution to the acquirement of speaking competencies and so it helps learners to become *independent users* of the language by the CEFR definition. So it is inevitable to acknowledge CLIL contribution to the improvement of language skills and competencies of learners.

[5] Common European Framework of Reference for Languages, abbr. CEFR. (http://www.coe.int/t/dg4/linguistic/Source/Framework_en.pdf)

2 PRACTICAL PART

2.1 Lesson Planning

In this chapter, we provide a closer look at designing our lesson plans, since CLIL approach is challenging not only for students but also for teachers. To make CLIL effective, in other words, to pursue the aims of this method and provide scaffolding, we found it inevitable to plan the lesson consistently.

Materials (Lesson plans 1 – 6 and their related worksheets or power-point presentations) present in this work were tested. In addition, untested materials which include Probability; the Pascal Triangle and Binomial Theorem are in *Appendices A-D.*

First step when planning a lesson was to sum up the objectives of the lesson by the guiding principles. When teaching via CLIL method, we divided objectives into three groups:

1) objectives related to the *mathematical content* – these state where CLIL lesson meets the standards of the curriculum of the subject;
2) objectives related to the different *types of language* – objectives achievable during the lesson which develop competencies of learners in English language;
3) *formative* objectives – related to the improvement of the personality of students.

We found some tasks which meet these objectives on the web pages and books which are mentioned in the Bibliography. Own tasks were introduced as well. The focus was on their linguistic part – to be approachable by students, and content – to be compatible with standards.

It was important that students were able to carry out tasks individually or with the help of a classmate or the teacher. Having wanted to create stimulating environment for language learning, students were given opportunity to produce language while working in pairs or in groups. In order to solve tasks, they had to communicate, cooperate and exchange information. In doing so, students enjoyed advantages of this method. At this stage, noteworthy role of the teacher was to monitor students. Students were expected to learn language *for* learning introduced during the lesson, so that they were able to keep up with the pace of the following lesson.

To improve thinking and learning of learners, students were asked to come up with rules or principles how to solve tasks. These could be applied to similar problems.

Guiding questions provided by the teacher were helpful in their reasoning. This type of questions is present in lesson plans we have prepared.

We are aware of the fact that the teacher should pay particular attention to the language since it is a vehicle for learning. In our lesson plans, the language crucial for students is always specified. The way of its presentation is not mentioned – in general, vocabulary items are explained through descriptions (e.g. *odd numbers* are one, three, five and so on), pictures and demonstrations. Translation of the word is used only when it is inevitable. However, since our aim was to make students bilingual, mathematical terms were introduced in Slovak language as well as in English language.

Moreover, we considered brain teasers to be suitable as an introduction to the CLIL lessons – students needed to think to solve the task and use English to explain their solution. These are suitable as language showers in all types of CLIL – from partial to total immersion.

To be able to understand our lesson plans, readers should keep in mind that

- each lesson is divided into stages which help to keep logical order of the lesson;
- timing is just informative;
- common abbreviations used in lesson plans are: T = teacher; S(s) = student(s), V = language for learning; E = English; WS = worksheets.

Lesson plans which were tested are followed by the evaluation (Lesson plans 1 – 6 and their related worksheets or power-point presentations).

2.2 Lesson Plans

Teaching materials comprising the theme Combinatorics were tested in the Gymnázium Jozefa Gregora Tajovského (abbr. GJGT) among the second grade students of the class with extended mathematical education. The mathematical content corresponds to the standards these students were supposed to reach.

Lesson plans and their related presentations elaborating Statistics were tested among the students of the third grade of Katolícke gymnázium Štefana Moysesa (abbr. KGŠM).

Unverified materials elaborating the theme Probability; the Pascal Triangle and Binomial Theorem are in *Appendices A-D*, as it is mentioned above.

Lesson plan 1 – **Introduction to Combinatorics**

Subject:	Mathematics	**Grade**:	2nd graders
Theme:	Binomial Coefficient	**Time**:	45 min

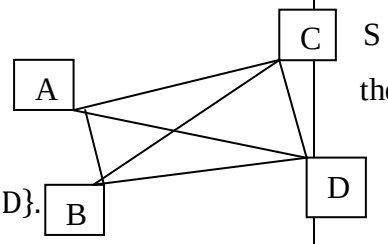
Content objectives	Students will be able to • simplify expressions which contain factorials; • determine the number of combinations that can occur for ***n*** things taken ***k*** at a time; • use the multiplication principle to determine in how many ways two or more events can occur together.
Language objectives	Students will be able to • follow instructions provided in English; • describe solutions in English language using vocabulary introduced during the lesson**.**
Formative objectives	Students will be able to • cooperate during group/pair work.

Language for learning: combination, principle, multiplication principle, factorial, arrangement, In how many different ways...?, times, be equal to, product, evaluate, the smallest value, set, subset, formula

Materials: papers with Language for Learning, worksheets

Stage & Time	**Procedure**	**Notes**
Introduction (4 min)	Introduction to the lesson using CLIL method. What is the reason we use (Slovak or English) language? Why do we speak? Why do we learn English language? *To communicate, to express ideas, to improve mutual understanding between nations....* T revises Ss´ ideas. Aim of the lesson: to use English as a means of communication. If you want to be educated / clever what should you	*pair work* Ss realize the pros and cons of learning in English the code

	do? *Listen to a T, study, do not disturb others,...* T: I hope that together we will improve a little bit. We are going to learn English through Mathematics and Mathematics through English language. If you have any questions, do not hesitate and ask.	of conduct
Warm up (3 min)	**Binomial coefficients** Four people stand in front of the class. Their aim is to make a line. 1 person per 1 place. Choose the person who will occupy the first, the second, the third and the last position. In how many different ways can they occupy the first place? *There are 4 different ways to occupy the first place*. In how many different ways can be the second place occupied? *There are 3 different ways to occupy the second place*. 4 choices — 3 choices — 2 choices — 1 choice 1^{st} place — 2^{nd} place — 3^{rd} place — 4^{th} place $4 \cdot 3 \cdot 2 \cdot 1 = 24$ What is the symbol for the **product** of all the numbers from n down to and including 1? *(n!)*	4 Ss in front of the class **V: times, be equal to, product** T reads aloud the equation and points out to algebraic terms
Presentation (2 min)	**Factorial** The **product** of all the numbers from $\boldsymbol{n}$ down to and including 1 is called $\boldsymbol{n}$ factorial. The symbol for $\boldsymbol{n}$ factorial is $\boldsymbol{n!}$ $\boldsymbol{n \in N}$: $0! = 1$ $n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \ldots \cdot 3 \cdot 2 \cdot 1 = n!$ $n! = n(n-1)!$ and $n!$ is the factorial of n.	T distributes WS S reads the table
Practice	Ss work individually and then T and Ss check solutions. **Exercise 2 A)** 5, **B)** a) 3!, b)4!, c)5!, d) 6!	*individual work*

(5 min)	**V: evaluate, the smallest value**	
Motivation (2 min)	T: I feel like having fruit salad. What fruit can I put in there? *ex. 'My fruit salad is **combination** of apples, grapes and bananas'* . Would it make difference if I put apples first, then bananas and then grapes? *The order of fruits are in is not important - it's the same fruit salad.* **Combination** – the order is <u>not</u> important. **GRAMMAR –** fruit/fruits; fish/fishes T: *Fruit* is usually uncountable (*Fruit is inexpensive here*). It is used as countable mainly to refer to one or more types of fruit (*oranges and other citrus fruits).*	Ss name some fruit
Presentation (5 min)	**k-combination without repetition of a set *S*** A *k*-combination without repetitionof a set *S* is a **subset** of *k* distinct elements of *S*. If the set has *n* elements the number of *k*-combinations is equal to the binomial coefficient: $\binom{n}{k} = \frac{n!}{(n-k)!k!}$ *(the formula)* where *n* is the number of things to choose from, and you choose *k* of them. (No repetition, order doesn't matter.) Give me an example of the Binomial coefficient.	S reads the table **V: set, subset, formula** formula remains written on the board
Practice (3 min)	**<u>Exercise 3</u>** Ss draw 4 points $\{A, B, C, D\}$ and each handshake illustrate as a line. Namely $\{A, B\}, \{A, C\}, \{A, D\}, \{B, C\}, \{B, D\}, \{C, D\}$.  There are $\binom{4}{2} = 6$ combinations of two elements out of the set of 4 elements.	S reads the text
Presentation (2 min)	**GRAMMAR**: Collective nouns Each <u>couple shake</u> a hand. / Each <u>couple shakes</u> a hand.	

	Collective nouns such as: *army, herd, family, jury,...* can be followed by a singular or plural form.	
Practice (5 min)	**Exercise 4** a) $\binom{n}{1} = n$ (the number of one-element subsets of a set containing n elements); $\binom{n}{1}$ predstavuje počet jednoprvkových podmnožín množiny, ktorá má n prvkov. Je zrejmé, že týchto jednoprvkových kombinácií je n; b) $\binom{n}{0} = 1$ (each set has only one empty subset); c) $\binom{0}{0} = 1$; d) $\binom{n}{n} = 1$ (the number of *n*-element subsets of a set containing *n*-elements). T demonstrates using 3 items- a pen, pencil & pen case. There is **the only way** to create a group of 3 and it is that we take all the objects. If we write it down through formula: $\binom{n}{n} = \frac{n!}{n!0!} = 1$. e) $\binom{n}{k} = \binom{n}{n-k}$ It means that $\binom{5}{2} = \binom{5}{3}$, $\binom{100}{3} = \binom{100}{97}$.	*pair work* Ss evaluate binomial coefficients using the formula Ss describe solutions
Motivation (4 min)	**Exercise 5** Students compare 5a), 5b). From these problems make a principle how to solve counting problems where there are several levels of choices to be made.	*pair work*
Presentation (2 min)	**THE MULTIPLICATION PRINCIPLE** T: forming the line (warm up activity) of Ss illustrates this principle. n_1 choices — n_2 choices — n_3 choices — n_4 choices 1st stage — 2nd stage — 3rd stage — 4th stage The number of all possibilities is $\boldsymbol{n_1 \cdot n_2 \cdot n_3 \cdot n_4}$.	*lockstep*
Practice (5 min)	**Exercise 6** She has to make 3 choices. $\binom{5}{1}$ choices — $\binom{8}{1}$ choices — $\binom{12}{1}$ choices skirt — blouse — shoes	*pair work* Ss describes

<table>
<tr>
<td></td>
<td>$\binom{5}{1} \cdot \binom{8}{1} \cdot \binom{12}{1} = 480$
She can wear 480 different outfits.
<u>Exercise 7</u> In how many different ways...?
a) Since there are no restrictions, we simply want to arrange 7 people in a line, and this is done in 7! = 5040 ways.
b) Place 2 boys first and then arrange the remaining 5 people. There are 4 choices for the first place. There are 3 choices for the last place. There are 5! choices to arrange the remaining 5 people. There are $4 \cdot 5! \cdot 3 = 1440$ ways to perform the arrangement.</td>
<td>their solution</td>
</tr>
<tr>
<td>Conclusion
(2 min)</td>
<td>Kombinatorika sa od všetkých matematických disciplín, s ktorými sa v škole zoznamujete, líši v niekoľkých smeroch:
1) kombinatorika sa zaoberá len vlastnosťami konečných množín (koľko bodov má trojuholník / priamka?),
2) vo väčšine prípadov nemáme možnosť overiť si správnosť výsledku a preto v kombinatorike viac než inde platí, že cvičenie robí majstra – ako je to po anglicky?
<u>Brain Teasers</u>
1) It was still Mount Everest.
2) The only place in the world where you can look in all directions, and have them all be south, is at the North Pole. So the bear must be white.
<u>Exercise 8</u>
<table>
<tr><th>We write these expressions</th><th>We read these expressions in English</th></tr>
<tr><td>$\frac{4!}{5!}$</td><td>four factorial over five factorial</td></tr>
<tr><td>$\binom{4}{3}$</td><td>four choose three</td></tr>
<tr><td>5 x 6 = 30</td><td>five times six is equal to thirty</td></tr>
<tr><td>$\{A, B, C, D\}$</td><td>a set containing 4 elements A,B,C,D</td></tr>
</table>
</td>
<td>lockstep

<u>HW:</u>
Complete the WS. Find answers to the Brain Teasers.</td>
</tr>
</table>

Language for Learning

symbol	Operation	Verb	how to read the operation	result
+	Addition	to add	plus	sum
−	Subtraction	to subtract	minus	difference
·	Multiplication	to multiply	multiplied by / times	product
÷	Division	to divide	divided by	quotient

$7 \cdot 4 = 28$	seven times four <u>is equal to</u> twenty-eight
here 7 and 4 are the '**factors**'	seven times four <u>equals</u> twenty-eight
and 28 is the '**product**' .	seven times four <u>is</u> twenty-eight

$\frac{8}{5}$ We can read this **fraction** as *eight-fifths*, *eight over five*, or *eight divided by five*.

evaluate	vyčísliť
the smallest value	najmenšia hodnota
formula	vzorec
element	prvok
set	množina
subset	podmnožina
disjoints sets	disjunktné množiny
double figures	dvojciferné čísla

Useful phrases

What does ... mean?	Čo to znamená?
How do you spell ...?	Ako vyhláskujete...?
In how many ways ...?	Koľkými rôznymi spôsobmi...?
The order doesn't matter.	Na poradí nezáleží.
I want to explain that ...	Chcem vysvetliť, že...
First of all …	Najprv...
In other words …	Inými slovami...
Consequently …	Následne...
The result is …	Výsledok je...
We shall prove...	Ukážeme, že ...
Thus ..	Preto

The key idea in answering this question is to consider...
Kľúčovou myšlienkou pri riešení je uvažovať....

Thus, by the multiplication principle the number of different ways of choosing ... is ...
Preto, podľa pravidla násobenia, počet rôznych spôsobov akými môžeme vybrať.... je...

Worksheet 1 – Introduction to Combinatorics

'Practice makes perfect.' (Cvičenie robí majstra.)

Brain Teasers

1) What was the highest mountain in the world before Mount Everest was discovered?

_____

2) A man builds a house rectangular in shape. All the sides have southern exposure. A big bear walks by. What colour is the bear? Why? _____

_____

Exercise 1 Fill gaps in *yellow tables* and in *Exercise 7* with these words:

factorial	**subset**	**In how many different ways**	**multiplication**	**product**

The _____ of all the numbers from n down to and including 1 is called n **factorial**. The symbol for n factorial is $n!$ $n \in N$:

$$\boldsymbol{n! = n(n-1)(n-2)(n-3)\ldots 3 \cdot 2 \cdot 1} \qquad 0! = 1$$

Exercise 2

A) What is the smallest value of n for which: $n! > 100$?

B) Write as factorials:

a) 6 = b) 24 = c) 120 = d) 720 =

A k-**combination without repetition** of a **set** S is a_____ of k distinct elements of S. If the set has n elements the number of k-combinations is equal to the b**inomial coefficient**:

$$\binom{\boldsymbol{n}}{\boldsymbol{k}} = \frac{\boldsymbol{n!}}{\boldsymbol{(n-k)!k!}} \qquad \textit{(the formula)}$$

where n is the number of things to choose from, and you choose k.
(No repetition, order doesn't matter.)
kombinačné číslo

$\binom{n}{k}$ *'n choose k'*
„en nad ká"

$\binom{4}{2}$ *'4 choose 2'*
„štyri nad dvomi"

Kombinácia bez opakovania k - *tej* triedy z n prvkov **množiny** S je ľubovoľná k-prvková **podmnožina** množiny S *(prvky sa neopakujú)*.

Exercise 3 Four friends A, B, C, D are standing in a circle. Each couple shake a hand. How many handshakes will occur?

Exercise 4 Let $n, k \in N,\ k \leq n$. Evaluate each of the following using the formula and then provide an explanation: $a)\ \binom{n}{1} =$ $b)\ \binom{n}{0} =$ $c)\ \binom{0}{0} =$

$d)\ \binom{n}{n} =$ $e)\binom{n}{n-k} =$

Exercise 5 Count the number of ways that customers at Kent's Deli can construct a sandwich. They have a choice of three types of meat - pork, chicken or beef. They also have a choice of two types of bread.

a) draw a diagram by choosing bread first then meat second,
b) draw a diagram by choosing meat first then bread second.

The_____ principle (pravidlo súčinu). Suppose that a choice is to be made in k stages. If there are $\boldsymbol{n_1}$ choices for the 1st stage and $\boldsymbol{n_2}$ choices for the 2nd stage,...., and $\boldsymbol{n_k}$ choices for the k-th stage then there are $\boldsymbol{n_1 \cdot n_2 \cdot \ldots \cdot n_k}$ choices altogether.

Exercise 7 Four boys and three girls are to be seated from left to right in a row. _____ _____ can this be done:

a) without restriction?
b) if there is a boy at each end of the row?

Exercise 8 Complete the table:

We write these expressions:	**We read these expressions in English:**
	four factorial over five factorial
$\binom{4}{3}$	
$5 \cdot 6 = 30$	
	a set containing 4 elements A,B,C,D

Evaluation of the lesson 1 – **Introduction to Combinatorics**

Date: 28th May, 2012 Class: 2nd graders GJGT

Level of proficiency in English: A2 – B1 according to CEFR

Lesson plan and Worksheet

The lesson plan helped me to keep activities in a logic order. I was able to stick to it. Students solved the tasks and answered all the guiding questions related to the cognitive demands, so I think that tasks were at the appropriate level and easily accessible by all the learners. Worksheet helped learners to follow the course of the lesson and supported language. I think that we achieved the objectives.

My teaching

It was first experience of these learners with the CLIL method. The enthusiasm of the students, their interest and curiosity exceeded my expectations. Students particularly enjoyed brain teasers – it was their first experience with the integration. It created good working atmosphere. Students gave me undivided attention since the following course of the lesson was quite unpredictable to them. I put an emphasis on the development of the listening skills, so learners could adapt to new learning conditions. However, learners could have been given more opportunities to speak. To keep learners motivated, I paid attention to the clarity of the instructions and comprehension of the learners.

The learners

I had taught these students (in Slovak language) before the education via CLIL took part, so it seemed that students felt comfortable during the lesson.

I have observed that many of these students are individualists so they found it difficult to cooperate. Nevertheless, I insisted on the promotion of the cooperation rather than competition so hopefully this lesson contributed to the change of classroom culture.

Sometimes, it was difficult to get all the students involved in the production of the language. This is the result of the variety of the level of efficiency in English between learners. I noticed that less confident learners felt insecure when I asked them even simple questions. They were asked to provide reasoning of the solution in Slovak language. However, especially the handout Language for learning contributed to their later confidence in speaking.

Lesson plan 2 – **The Multiplication and Counting Principles**

Subject:	Mathematics	**Time**:	45 min
Theme:	The Multiplication and Counting Principle	**Grade**:	2nd graders

Content objectives	Students will be able to • determine the number of combinations that can occur for $\boldsymbol{n}$ things taken $\boldsymbol{k}$ at a time; • use the multiplication principle and the counting principle to determine in how many ways two or more events can occur together.
Language objectives	Students will be able to • follow instructions provided in English; • describe their solutions correctly using mathematical language; • use first conditional sentence.
Formative objectives	Students will be able to • cooperate during group/pair work.

Language for learning: double figures, disjoint sets, via, at least, under each of the following condition, be assigned to

Materials: worksheets

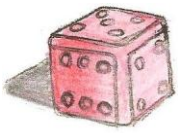
Stage & Time	**Procedure**	**Notes**
Introduction Warm up (3 min)	Introduction. **Brain Teasers** 1) *Just once. Because, the number is now 30 and not 33.* V: **to subtract:** T writes on the board 30-3=27. 2) *There are more Chinese men than Japanese men.*	*pair work*
Revision (3 min)	Čo je to kombinácia *k* - tej triedy z *n* prvkov množiny M? *Kombinácia k - tej triedy z n prvkov množiny M je ľubovoľná k-prvková podmnožina n-prvkovej množiny M.* What is a *k*-combination of a set *S* containing	*lockstep*

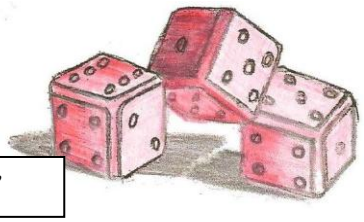
	n elements? *A k-combination of a set S is a* ***subset*** *of k distinct elements of S.* How would you read $\binom{10}{2}$ in English and in Slovak? Can you give me some examples of **binomial coefficient**? *English: ten choose two, Slovak: desať nad dvomi.* Write down the formula using factorials to express the binomial coefficient. $\binom{n}{k} = \frac{n!}{(n-k)!k!}$ How do we call binomial coefficient in Slovak language? *Kombinačné číslo.*	S comes to the board and writes down the formula Checking HW – Brain Teasers.
Practice (7 min)	**Exercise 1** The key idea in answering this question is to consider the problem in stages. At the first stage – from Ashbury to Brampton - there are two choices. For each of these choices, there are three choices to make at the second stage - from Brampton to Carmichael. Thus, the number of different routes is $\binom{2}{1} \cdot \binom{3}{1} = 6$. **Exercise 2** Since $\binom{2}{1} \cdot \binom{6}{1} \cdot \binom{32}{1}$= 384, there are 384 different outcomes of this experiment.	*pair work* then a couple presents the solution V: **via** V: **a die**
Motivation (5 min)	**Exercise** 3 All double figures can be divided into two **disjoints** sets: • 1st set contains those double figures whose digits differ. Let *p* be the number of those double figures; • 2nd set contains those double figures whose digits are the same (11, 22, ...99). How many elements belong to this (2nd) group? (9) How many double figures exist? *There are 90 double figures altogether.* It means: *p+9=90.* Thus there are 81 double-digit numbers which digits differ. Think of another way how to solve this problem. (*Some students can use ordering*).	V: **double figures, disjoint sets -** if two sets have no element in common they are called **disjoint**

Presentation (3 min)	How do we find out the number of elements of two disjoint sets? **The Counting Principle** If $\boldsymbol{A_1}$, $\boldsymbol{A_2}$,... $\boldsymbol{A_3}$,..., $\boldsymbol{A_n}$ are disjoint finite sets and $\boldsymbol{A_1}$ contains $\boldsymbol{p_1}$ elements, $\boldsymbol{A_2}$ contains $\boldsymbol{p_2}$ elements,..., $\boldsymbol{A_n}$ contains $\boldsymbol{p_n}$ elements then the number of elements of the set $\boldsymbol{A_1} \cup \boldsymbol{A_2} \cup ... \cup \boldsymbol{A_3}$,... $\boldsymbol{A_n}$ is $\boldsymbol{p_1}+\boldsymbol{p_2}+...+\boldsymbol{p_n}$.	
Practice (10 min)	**Exercise 4** The student can answer 8 out of 10 questions or 9 out of 10 questions or 10 out of 10 questions. Thus, by the counting principle the number of ways of answering at least 8 out of 10 questions on exam is: $\binom{10}{8}+\binom{10}{9}+\binom{10}{10}= 56.$ **Exercise 5** a) $\binom{20}{5} = 15504;$ b) $\binom{12}{5} \cdot \binom{8}{0} + \binom{12}{4} \cdot \binom{8}{1} + \binom{12}{3} \cdot \binom{8}{2}$=10 912. **Exercise 6** Since $\binom{7}{1} \cdot \binom{6}{2} \cdot \binom{4}{3} \cdot \binom{1}{1} = 420$, there are 420 different ways how students can be assigned to these rooms. **Exercise 7** $\binom{n}{2} = \frac{n!}{2!(n-2)!} = 10$. Answer: n =5.	*pair work* then a couple presents the solution V: **at least** V: **under each of the following condition, be assigned to**
Motivation (3 min)	Why $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$? **Hint**: Let us have the group of 6 boys. One of them is Peter. In how many different ways can we choose 3 boys out of 6 boys? *There are* $\binom{6}{3}$ *combinations.* How many different combinations of 3 boys are there, if Peter <u>is one</u> of them? *There are* $\binom{5}{2}$ *combinations.* How many different combinations of 3 boys are there, if Peter <u>is not</u> there? *There are* $\binom{5}{3}$ *combinations.* If we have any group of 3 boys, there are just two options: Peter is there or he is not there. So the total number	*lockstep*

	of combinations of 3-boys group equals: $\binom{6}{3} = \binom{5}{2} + \binom{5}{3}$.	
Presentation (9 min)	**<u>Challenge task</u>** T: Let $A = \{a_1, a_2, \dots, a_{n+1}\}$. A contains $n+1$ elements. Let´ s make $(k+1)$-element subgroups. What is the number of (k+1)-element subsets of a set A? $\binom{n+1}{k+1}$. T: Let´s divide A into two groups. Those which contain the element a_{n+1} will be in <u>the first group</u> and those which do not contain this element will be in <u>the second group</u>. **<u>STEP 1</u>** Let´s deal with <u>the first group</u>. How many different (k+1)-element subgroups are there which contain a_{n+1}? *The first group will be made this way: we add to all possible k-combinations of a set $S = \{a_1, a_2, \dots, a_n\}$ the element a_{n+1}. The number of these combinations is $\binom{n}{k}$.* **<u>STEP 2</u>** Let´s deal with *the second group.* How many different (k+1)-element subgroups are there which do not contain a_{n+1}? *The second group contains $\binom{n}{k+1}$ different combinations.* The total number of (k+1)-element subsets of a set A is $\binom{n+1}{k+1}$. Thus $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$. These exercise illustrates the counting principle.	*pair work* T helps Ss to find solution T or a S presents the proof
Conclusion (2 min)	Translate ‘binomial coefficient’ into Slovak language. *Kombinačné číslo.* What are two major combinatorial principles? *The multiplication principle and the counting principle. (Kombinatorické pravidlo súčinu a súčtu).*	*lockstep* HW: complete the worksheet

Worksheet 2 –THE MULTIPLICATION AND COUNTING PRINCIPLES





'The die has been cast.'

Brain Teasers

1) How many times can you subtract the number 3 from the number 33?

_____

2) Why Chinese men eat more rice than Japanese men do?

_____

Fill the blank gaps marked as ☐ with words in bold from the purple box.

Exercise 1 Suppose that three towns, Ashbury (A), Brampton (B), and Carmichael (C), are located in such a way that two roads connect A to B and three roads connect B to C. How many different routes can one take to travel from A to C ☐?

Exercise 2 A coin is tossed, a ☐ is rolled, and a card is drawn from a pack. There are 32 cards in the pack. How many possible outcomes does this experiment have? (Note: a coin has a tail side and a head side.)

Exercise 3 How many ☐ are there, which consists of two different figures?

die – pl. **dice**
condition
double figures – double digits
via – travelling through a place on the way to another place
at least – not less than any particular amount of money
assigned to

The Counting Principle

If $A_1, A_2, \ldots, A_n$ are disjoint finite sets and A_1 contains p_1 elements, A_2 contains p_2 elements, ..., A_n contains p_n elements then the number of elements of the set

$A_1 \cup A_2 \cup \ldots \cup A_n$ is $p_1 + p_2 + \ldots + p_n$.

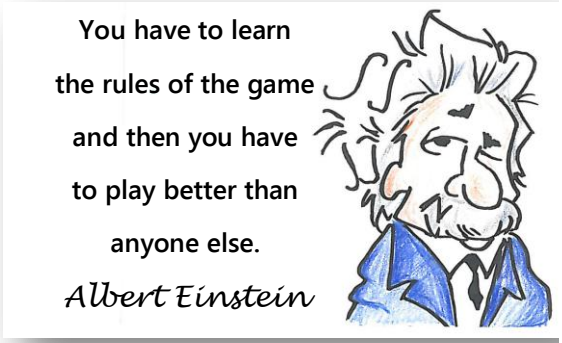
Exercise 4 A student is asked to answer 8 out of 10 questions on exam. In how many different ways can he choose the questions?

Exercise 5 A class has 20 students, of which 12 are females and 8 are males. In how many different ways can a committee of 5 students be picked from this class **under each of the following**

a) No restriction is placed on the number of males or females on the committee.
b) No more than 2 males are to be included on the committee.

Exercise 6 When seven students take a trip, they find a hotel with three rooms available – a room for one person, a room for two people, and a room for three people. In how many different ways can the students be these rooms? (One student has to sleep in the car.)

Exercise 7 A group of friends have reserved a tennis court. They find that there are ten different ways in which two of them can play a single game on this court. How many friends are in this group?



A Challenge task: Prove: $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$ (hint: use the counting principle).

Evaluation of the lesson 2 – **The Multiplication and Counting Principles**

Date: 29th May, 2012 Class: 2nd graders GJGT

Level of proficiency in English: A2 – B1 according to CEFR

Lesson plan and Worksheet

I think that by following of the lesson plan, the set objectives are reachable. By the analysis of the content and cognition for potential difficulties when planning the lesson, I partly managed to overcome difficulties which occurred in the previous tuition.

The emphasis was on the mathematical content. Although the lesson plan and worksheet concentrated on the revision and practice, the progression was built into language and content tasks. I think that there was cohesion between our teaching aims and the learning outcomes.

My teaching

The focus of this lesson was mainly on the learner confidence-building and sense of achievements in Mathematics as well as in English. To support the progression of the learners, students recycled new language from the previous lesson. This lesson, my talking time was reduced and students were given more opportunities to produce language than in the previous one. Students were encouraged to use language creatively.

The most difficult part for me was to keep all the students involved. Students were not tolerant towards each other, they showed impatience when somebody had troubles with language or content. Worksheet helped me to cope with this problem – the skillful students were asked to solve other tasks. Then I was more secure in teaching and it resulted in unstrained working atmosphere.

The learners

Individual outcomes varied, but the participation of learners in activities was encouraged by a range of scaffolding learning tools. It seemed that even less proficient learners enjoyed the lesson. These students apparently paid attention to the language for learning introduced during the previous lesson, so it enabled them to easily follow the course of the lesson. Students who underestimated language, had difficulty to keep up with peers. It was a two-edged weapon – it resulted in the promotion of cooperation.

Lesson plan 3 – **Combinations, Variations**

Subject:	Mathematics	**Grade**:	2nd graders
Theme:	Combinations, Variations	**Time**:	45 min

Content objectives	Students will be able to • determine the number of combinations with repetition that can occur for $\boldsymbol{n}$ things taken $\boldsymbol{k}$ at a time; • determine the number of variations without repetition that can occur for $\boldsymbol{n}$ things taken $\boldsymbol{k}$ at a time.
Language objectives	Students will be able to • follow instructions provided in English; • use structure **be on the point of;** • to use structure **be to** to express future, instruction, order, prohibition.
Formative objectives	Students will be able to • cooperate during group/pair work.

Language for learning: variation, strategy, ID number, power of a number, root of a number, square of a number, square root of a number, be on the point of, be due to

Materials: worksheets, boxes with safety-matches

Stage & Time	Procedure	Notes
Introduction Warm up (4 min)	Introduction. **NIM**- Ss work in pairs. Each pair has 16 safety-matches in row. Ss take turns. Each S can take 1 up to 3 matches when it is her/his turn. The person who takes the last match/matches wins!  There is a strategy which secures that player who starts later, always wins. What is the strategy? *If the first player takes 3 matches you have to take 1 match (so the sum will be 4).*	*pair work* V: **strategy**

Revision (2 min)	Čo je to kombinácia *k* - tej triedy z *n* prvkovej množiny M? *Kombinácia k - tej triedy z n prvkovej množiny M* je ľubovoľná *k*-prvková podmnožina *n*-prvkovej množiny *M*. What is a *k*-combination of a set *S* containing *n* elements? *A k-combination of a set S is a subset of k distinct elements of S.* In real world, where is combinatorics helpful? *It is helpful when we design efficient ways to transmit data on the internet.* Write down the formula using factorials to express the binomial coefficient. $\binom{n}{k} = \frac{n!}{(n-k)!k!}$. How do we call binomial coefficient in Slovak language? *Kombinačné číslo.*	*lockstep*
Motivation (4 min)	**Combination with repetition** **<u>Exercise 1</u>** Let be the first type of the bottle *A*-type. Let be the second type of the bottle *B*-type. What are different **combinations** of choosing the bottle? *There are the following combinations of choosing the bottle:* $\{A, B\}, \{A, A\}, \{B, B\}$. Can I use $\binom{n}{k}$ to solve this problem? *No.* Why? *Because in this case we can <u>repeat</u> the elements.*	theme *lockstep*
Presentation (5 min)	There are two types of combinations (remember the order does **not** matter): 1) **repetition is allowed**: such as coins in your pocket (2,2,1,1,1) 2) **no repetition**: such as lottery numbers (2,14,15,27,30,33).	S reads definition ***C´*** (WS)
Practice	**<u>Exercise 2</u>** The possible choices are combinations with repetition of three elements. Since we choose	*pair work*

(5 min.)	5 elements, the number of different choices is C'(3,5) = 21. Why did we need to know that amount of each kind of sweets? *We know that we have got as many sweets of any kind as we need.* **Challenge**: We have to subtract only the one option – choosing 5 red sweets. So there are 20 choices. **Exercise 3** Using the formula we obtain $C'^{5}_{2} = \binom{5+2-1}{2} = \frac{(5+2-1)!}{2!\,(5-1)!}$. By the formula there are 15 different products.	
Motivation (4 min)	A woman has individual photos of each of her three children – Mary, Scott, and Joe. In how many different ways can she arrange these photos in a row on the desk? *From previous discussion, we know there are* $3 \cdot 2 \cdot 1 = 6$ *possible outcomes. Each of these arrangements is different. The order of the pictures is important. When we have a group of things arranged in a definite order, we have a variation.*	*lockstep*
Presentation (4 min)	**ORDERING** (variations without repetition)	S reads definition WS
Practice (10 min)	**Exercise 4** There are $5 \cdot 4 = 20$ different arrangements. **Exercise 5** 4! ways (one person has the same position all the time and we arrange the rest of them). **Exercise 6** There are $24 \cdot 10 \cdot 10 = 2400$ possibilities of different IDs. It means that we cannot use this scheme. What is an ID number? **Identification number** is used by the governments of many countries as a means of tracking their citizens (for the purposes of work, taxation, government, government benefits etc.).	*individual work* then presentation of solution

<table>
<tr>
<td>Presentation

(4 min)</td>
<td>GRAMMAR - be to structure
<table>
<tr><td>Three tablets to be taken twice a day.</td><td>instruction</td></tr>
<tr><td>The queen is to visit Portugal in November. (má navštíviť)</td><td>official arrangements</td></tr>
<tr><td>My doctor says I´m not to eat meat.</td><td>prohibition</td></tr>
<tr><td>He is to stay here till we return.</td><td>order</td></tr>
</table>
Other similar expressions: be about to, be on the point of (immediate future), be due to (timetables)
The BA11 is due to arrive from Paris at 1:10 (má priletieť).</td>
<td>pair work</td>
</tr>
<tr>
<td>Practice

(2 min)

Production</td>
<td>Translate into English:
• Pápež má v decembri urobiť veľké rozhodnutie. The pope is to make a big decision in December.
• Svadba sa má uskutočniť v sobotu 28. The wedding is to take place on Saturday the 28th.
Produce own sentences using the expressions: be about to, be on the point of, be due to.</td>
<td>lockstep</td>
</tr>
<tr>
<td>Conclusion

(1 min)</td>
<td>ORDER
– is important – variations (arrangement)
– is not important – combinations (choosing)</td>
<td>T writes on the board</td>
</tr>
</table>

Worksheet 3 – **Combinations, Variations**

Exercise 1 There are two different **types** of bottles in a wine cellar. In how many different ways can two bottles be chosen from the cellar? ($\{A, B\}$ and $\{B, A\}$ is still one way.)

A *k*-**combination with repetitions** from a set *S* is given by a sequence of *k* not necessarily distinct elements of *S*. The number of such combinations is given by:

$$C'^{n}_{k} = \binom{n+k-1}{k} = \frac{(n+k-1)!}{k!\,(n-1)!}$$

where $\boldsymbol{n}$ is the number of things to choose from, and you choose $\boldsymbol{k}$ of them.

KOMBINÁCIA s opakovaním k-tej triedy z *n* prvkov je neusporiadaná *k*-tica zostavená z týchto prvkov (prvky sa môžu opakovať)

Exercise 2 How many different products of two factors can be expressed by the multiplication of the numbers: 2, 3, 5, 7, 11?

Exercise 3 You have a box with 5 red sweets, a box with 10 yellow sweets and a box with 7 black sweets. In how many different ways can you choose 5 sweets? (**Challenge**: solve the situation if you have only 4 red sweets.)

k-členná VARIÁCIA bez opakovania z *n* prvkov je usporiadaná *k*-tica zostavená z týchto prvkov tak, že každý sa v nich vyskytuje najviac raz.

If a set has *n* elements, then a **variation without repetition** is the ordering of *k* objects if any object cannot be chosen more than once. The number of these variations is

$$V(n, k) = \frac{\boldsymbol{n!}}{(\boldsymbol{n-k})!} = \boldsymbol{n(n-1)(n-2)\ldots(n-k+1)}.$$

where *n* is the number of objects from which you can choose and *k* is the number of objects we can choose.
(repetitions are not allowed)

Exercise 4 If we are given the digits 1,2,3,4 and 5, how many two-digit numbers can we form if the digits cannot be repeated?

Exercise 5 In how many different ways can 5 people be arranged in a circle?

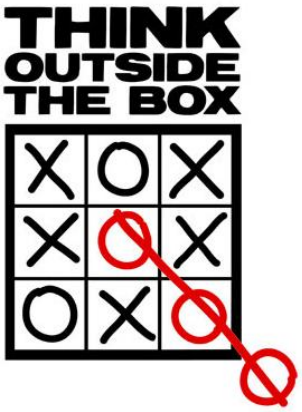
Exercise 6 A company has 2844 employees. Each employee *is to be given* an **ID number** that consists of one letter (out of 24) followed by two digits. Is it possible to give each employee a different ID number using this scheme? Explain.

If a set has *n* elements, then a **variation with repetition** is the ordering of *k* objects if repetitions are allowed. The number of these variations is $V'(n,k) = n^k$ where *n* is the number of objects from which you can choose and *k* is the number of objects we can choose.

k-členná VARIÁCIA s opakovaním z *n* prvkov je usporiadaná *k*-tica zostavená z týchto prvkov (prvky sa môžu opakovať)

Exercise 7 Underline the structure "to be" in the following sentences. Match corresponding uses of this structure.

Three tablets to be taken twice a day.	prohibition
The queen is to visit Portugal in November.	order
My doctor says I´m not to eat meat.	official arrangements
He is to stay here till we return.	instruction



Similar expressions: ***be on the point of*** (immediate future)**,** ***be due to*** **(**timetables), _____ .

LANGUAGE for LEARNING

symbol	**name**	**operation**	**how to read for** $n = 2$	**how to read for** $n \geq 3$
$( )^n$	upper index	power of a number	'square of a number' / 'number squared'	'n-th power of a number' / 'number to the power of n'
$\sqrt[n]{\ }$	the surd	root of a number	'square root of a number'	'n-th root of a number'

Read**:** 4^3, $\sqrt[3]{8}$, 4.5^2=100

Evaluation of the lesson 3 **– Combinations, Variations**

Date: 30th May, 2012 Class: 2nd graders GJGT

Level of proficiency in English: A2 – B1 according to CEFR

Lesson plan and Worksheet

Careful lesson planning contributes to the development of learners and to the improvement of the teacher. Experiences with CLIL made this lesson planning easier than the previous ones. I was able to stick to the lesson plan.

The important role of the teacher – to monitor and to recognize needs of the learners – increases with the use of the CLIL method, since the content needs and language requirements of the learners are diverse.

My teaching

Attitude of learners towards the lesson changed – their behavior was much more natural than during the previous lessons. The learners proficient in English did not require the attention of others and less proficient learners expressed themselves more frequently and with spontaneity.

Self-confidence of the less proficient learners in English was built up. They realize that language is used to communicate, and making mistakes is a part of their learning. These students experimented with language. More confident learners critically examined the produced language and quite often they were helpful in reformulating ideas.

The learners

At this stage of the tuition, it seemed that learners got used to the CLIL method and felt comfortable throughout the lesson. Their expectations could meet the reality and it served as an important source of motivation which was so notable during the lesson. Learners seemed to enjoy the lesson and improve at the same time.

Students were fond of the game with matches. The mathematical content was quite challenging for them so they did not realize that they were improving their language skills. English became a matter of course – the first step to the spontaneous production as well.

Lesson plan 4 – **Revision, Permutations**

Subject: Mathematics **Grade**: 2nd graders

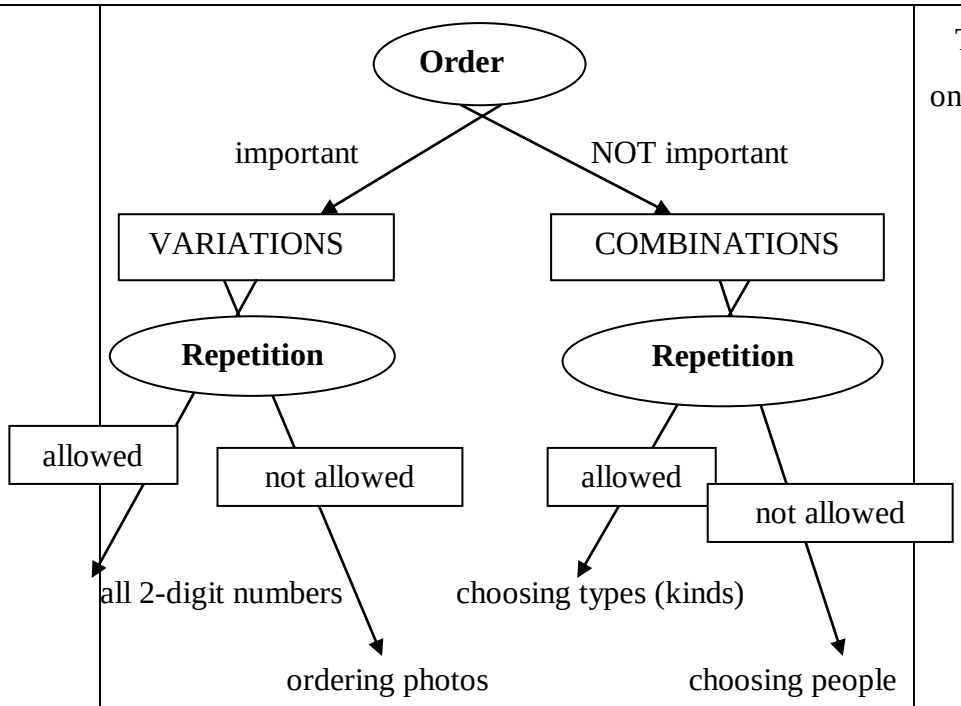
Theme: Revision of Combinatorics, Permutations **Time**: 45 min

Content objectives	Students will be able to • determine the number of variations with repetition that can occur for ***n*** things taken ***k*** at a time (things can/cannot be repeated); • determine the number of permutations with (and without) repetition.
Language objectives	Students will be able to • follow instructions provided in English; • use words such as **palindromes, anagrams, distinct, permutation** in their speech.
Formative objectives	Students will be able to • cooperate during group/pair work.

Language for learning: permutation, odd/even number, positive integers, palindromes, anagrams, distinct

Materials: worksheets

Stage & Time	**Procedure**	**Notes**
Introduction Warm up (4 min)	Introduction. **Brain Teasers** 1) 3+4=7 2) Keep the first bulb switched on for a few minutes so it gets warm. Then switch it off, switch another one on, walk into the room with bulbs, touch them and tell which one was switched on as the first one (the warm one) and the others can be identified easily.	distribute WS *lockstep*
Revision (3 min)	 In real world, where is Combinatorics helpful? *It is helpful when we design efficient ways to transmit data on the internet.*	*lockstep*

		T draws on the black board
Practice (10 min)	**Exercise 1** **a)** The number cannot begin with 0, so: There are 4 choices for the first digit. There are 5 choices for the second digit. There are 5 choices for the third digit. There are 5 choices for the fourth digit. The number of such four-digit numbers is $4 \cdot 5 \cdot 5 \cdot 5 = 500$ by multiplication principle. **b)** Since the number is greater than 8000, the first digit has to be 8 or 9. Then there are 2 choices for the first digit. There are 4 choices for the second digit. There are 3 choices for the third digit. There are 2 choices for the fourth digit. The number of such four-digit numbers greater than 8000 is $2 \cdot 4 \cdot 3 \cdot 2 = 48$. **c)** If the number is odd, it must end in either 7 or 9, but again it cannot begin with 0, so: There are 2 choices for the fourth digit. There are 3 choices for the first digit. There are 3 choices for the second digit. There are 2 choices for the third digit. The number of such four-digit odd numbers is $2 \cdot 3 \cdot 3 \cdot 2 = 36$.	*individual work* then S describes the solution & writes it on the board

	d) There are a number of ways to do this problem. The best solution is to realise that, of the 96 possible four-digit numbers made from these digits, each one will either be even or odd. We have already shown that there are 36 odd ones, so the number of even ones is 96 – 36 = 60. **Exercise 2** We will consider all mathematics books as 1 item, and all books of Physics as the other item. There are 4! different ways how to organize Mathematics books and 3! different ways how to arrange books of Physics. And we can arrange those two items in 2! ways. So altogether, there are $4! \cdot 3! \cdot 2! = 288$ different ways how to place books on the shelf. **Exercise 3** $24 \cdot 24 \cdot 24 \cdot 24 \cdot 1 \cdot 1 \cdot 1 = 331\,776$.	
Presentation (3 min)	**Permutation without repetition** – Ss read the definition (WS)	
Motivation (5 min)	**Exercise 4** Since $P(8) = 8! = 40\,320$; there are 40 320 anagrams of the word TRIANGLE.	
Motivation (4 min)	Consider the word ALL. How many different anagrams are possible? Can we use the formula for permutations with repetitions? What is the number of arrangements that can be made from these letters? (3 arrangements: ALL, LAL, LLA) How many kinds of letters are there? *(2 kinds – A& L)* How many letters A are there? (just one) How many letters L are there? (two) Where is the problem? There are two same letters L. If there would be L_1, and L_2, how many arrangements are there? (6) Let´s write down all the options for A, L_1, L_2: 1. AL_1L_2 2. AL_2L_1 3. L_1AL_2 4. L_2AL_1 5. L_1L_2A 6. L_2L_1A	*lockstep* T erases the indexes

	If we want to find the number of distinct arrangements of *n* things when *p* of the things are alike, then we divide *n!* by *p*!.	
Presentation (3 min)	**Permutation with repetition** – Ss read definitions (WS)	
Practice (10 min)	**Exercise 5** By the formula, there are: $\frac{11!}{1!4!4!2!}$=34 650 distinct arrangements. **Exercise 6** a) Since the order in which the people stand is important, we use variations. But we can't use the formula for variations directly. Since Jane and John insist on standing together, let's think of them as one object. Thus, we have 11 objects to arrange in a row and there are $P(11)$ways of doing this. For each of these arrangements, there are two ways of having Jane and John stand together - Jane-John or John-Jane. Thus, by the Counting Principle the total number of arrangements is $2 \cdot P(11) = 2 \cdot 11! = 79\,833\,600$. b) There are $P(12)$ ways of arranging the 12 people. Of these, $2 \cdot P(11)$ have Jane and John standing together [by part (a)]. All the rest have Jane and John standing apart. So the number of arrangements with Jane and John apart is $P(12) - 2 \cdot P(11) = 12! - 2 \cdot 11! = 399\,168\,000$. **Exercise 7** It is also a permutation problem (repetitions are allowed). A person has to travel 4 blocks East (E) and 4 blocks North (N). So a person must make a total of 8 moves (for example EENEENNN). Thus there are $P'(4,4) = \frac{8!}{4!4!} = 20$ different ways to go from A to B.	*pair work*
Conclusion (3 min)	Revision of formulas. HW: complete the worksheet.	*lockstep*

Worksheet 4 – **Revision, Permutations**

Brain Teasers

1) If one's company and two's a crowd what´s three and four? _____

2) There are three switches downstairs. Each corresponds to one of the three light bulbs in the attic. You can turn the switches on and off and leave them in any position. How would you identify which switch corresponds to which light bulb, if you are only allowed one trip upstairs?_____

_____

Exercise 1 How many different four-digit **positive integers** can be formed using the digits 6, 7, 8, 9 and 0

a) if there is no restriction and the numbers can be repeated?
b) without repetition if the number is greater than 8000?
c) without repetition if the number is **odd?**
d) without repetition if the number is **even?**

positive integers

odd number

even number

palindromes

anagrams

distinct

Exercise 2 Four mathematics books and three books of Physics are to be placed on a shelf. In how many different ways can this be done if the mathematics books are next to each other and the books of Physics are next to each other?

Exercise 3 How many different seven-letter **palindromes** are possible? (A *palindrome* is a string of letters that reads the same backward and forward, such as the string "*tatarratat*".)

Permutácia z *n* prvkov je každá *n*-členná variácia (bez opakovania) týchto prvkov.

A **permutation without repetition** of a set of distinct objects is an ordering of these objects.

The number of permutations of *n* objects is $P(n)=n!$.

Exercise 4 How many anagrams of the word *TRIANGLE* are possible? (An *anagram* of a word is a permutation of the letters of that word. For example, anagrams of the word *triangle* include *griantle*, *integral*, and *tenalgir*.)

n-členná permutácia s opakovaním z k prvkov je usporiadaná n- ica zostavená z týchto prvkov tak, že každý sa v nej vyskytuje aspoň raz.

PERMUTATION with repetition

If a set of n objects consists of k different kinds of objects with $\boldsymbol{n_1}$ objects of the first kind, $\boldsymbol{n_2}$ objects of the second kind, $\boldsymbol{n_3}$ objects of the third kind, and so on, where $\boldsymbol{n_1} + \boldsymbol{n_2} + \ldots + \boldsymbol{n_k} = n$, then the number of distinguishable permutations of these objects is

$$\mathbf{P}'(\boldsymbol{n_1}, \boldsymbol{n_2}, \ldots, \boldsymbol{n_k}) = \frac{\boldsymbol{n}!}{\boldsymbol{n_1}!\boldsymbol{n_2}!\ldots\boldsymbol{n_k}!}.$$

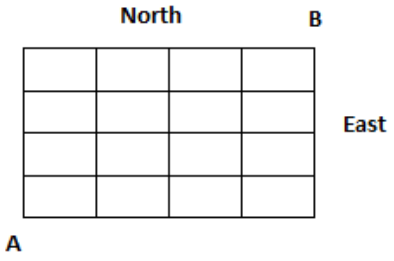
Exercise 5 How many distinct arrangements can be formed from all the letters of MISSISSIPPI?

Exercise 6 Twelve employees at a company picnic **are to** stand in a row for a group photograph. In how many ways can this be done if

a) Jane and John insist on standing next to each other?

b) Jane and John refuse to stand next to each other?

Exercise 7 The block diagram shown represents sixteen city blocks, with the lines indicating streets. A fire truck at point A wishes to travel to point B by travelling only North or East. How many different ways can the truck take?



Evaluation of the lesson 4 – **Revision, Permutation**

Date: 31st May, 2012 Class: 2nd graders GJGT

Level of proficiency in English: A2 – B1 according to CEFR

Lesson plan and Worksheet

The importance of the lesson was in comprehension of learners of the main Combinatorics principles and concepts – the content prevailed. The aim of the lesson was to provide the insight into the theme. By using designed activities and tasks from the lesson plan and worksheet we managed that teaching and learning outcomes met the aims of the lesson. I tried to ensure that learners progress cognitively, and to measure this progress, students were to take a test the next lesson to obtain summative assessment. Formative assessment was an essential part of each lesson. This lesson gave them the opportunity to self-evaluate the progress in the content and language.

My teaching

Having known the strengths and weaknesses of the students, I could adopt tasks to be solved at the appropriate cognitive level and by doing so, assist learners in their further development. It was the last lesson when teaching and learning via CLIL took part in this class. Having already experienced 'CLIL at work', I experimented with learners and teaching methods, which students quite enjoyed. Seeing that students were able to deduce the appropriate formula to solve the task, I think that we achieved the set objectives.

The learners

Most of the learners acquired language for learning from the previous lessons, so they had got the adequate vocabulary to answer my questions. It helped students to adopt positive approach to English language and the Mathematics. There were also students who did not overcome language barrier and were dependent on the translation. These students refused to speak English after the acknowledgement of the skills of their classmates in English. However, especially those students admitted that besides the conscious learning of the content and language, they learnt language unconsciously. The majority of students used the medium of the target language to reason their solutions.

Lesson plan 5 – **Introduction to Statistics**

Subject: Mathematics **Grade**: 2nd graders

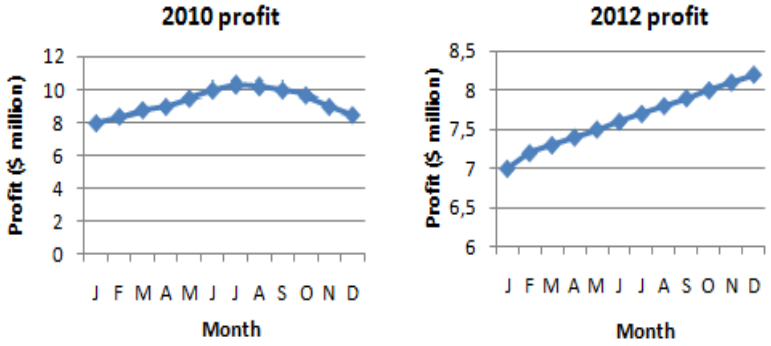
Theme: Mode, median, standard deviation **Time**: 45 min

Content objectives	Students will be able to • organize the set of given data; • represent the obtained results; • calculate the mode, the median, the mean and the standard deviation of the given set of data.
Language objectives	Students will be able to • follow instructions provided in English; • describe their solution using words introduced throughout the lesson; • use words like ‘data’ and ‘ information’ correctly.
Formative objectives	Students will be able to • cooperate during group/pair work; • tolerate each other.

Language for learning: mode, median, mean, standard deviation, graph, table

Materials: power-point presentation, papers with words

Stage & Time	**Procedure**	**Notes**
Introduction (4 min)	Introduction to the lesson using methodology CLIL. Ss realize the advantage of this kind of lesson so they can seize the opportunity to learn something new.	*slide 1*
Warm up (3 min)	How can we find out about the world around us? Students are given pieces of papers with the following words: 1) collect information/data, 2) organise information, 3) study the results, 4) represent the results.	*slide 2* **ordering**

	Their task is to put words in the logic order. **GRAMMAR**: ***information*** is an uncountable word and has only singular form. Use a singular verb after it.	
Motivation (3 min)	**Newspaper Article** In a newspaper article, the graphs shown below were accompanied by this text: 'Company profits looked much better in 2010 when compared with 2012'.  What is wrong with these pictures?	*slide 3*
Revision (5 min)	When we make statements about data that we have collected, we often want to • say which item is the most popular – **mode;** • which item is in the middle – **median;** • and which item is the average – **mean.** **GRAMMAR**: ***data* –** after data you can use a singular verb or a plural verb. Do not say '*a data*' or '*datas*'. Mode, median, mean. What do they tell us? • They represent different ways of trying to show **the centre of a data set.** • The type of data affects which you can use. The median can only be used for data that is *ordered.* **Mode** is the French word for *'fashion',* • and it is also the most fashionable (or most popular)	*slide 4* *slide 5* *slide 6*

<table>
<tr>
<td></td>
<td>value in a set of data;
• there may be more than one mode.
Median - the ‘middle value’ of a set when a set of values is arranged in order of their size.
Mean (or ‘average’) – is found by taking the sum of the values, then dividing the result by the number of values.</td>
<td>slide 7
Ss find the median
slide 8</td>
</tr>
<tr>
<td>Practice
(10 min)</td>
<td><u>Exercise 1</u> The canteen at school sells packs with frozen strawberries. This table shows the sales of packs with frozen strawberries per one week.
<table>
<tr><th>Day</th><th>Frequency</th></tr>
<tr><td>Monday</td><td>3</td></tr>
<tr><td>Tuesday</td><td>9</td></tr>
<tr><td>Wednesday</td><td>3</td></tr>
<tr><td>Thursday</td><td>30</td></tr>
<tr><td>Friday</td><td>8</td></tr>
</table>
A. Calculate the mode.
B. Calculate the mean.
C. What is the median?
• Since these packs takes too much room, the canteen staff wants to know how many pack they should have in the freezer at the beginning of the day. What is your piece advice?
• Think about the calculations you have made. Would the mode, mean or the median be a good value to rely upon?
Answers: Mode: 3; mean: 10.6; median: 8.5.
(The mean will sometimes give you information that is not very useful.)
<u>Exercise 2</u> The mean of a set of four numbers is 10. The numbers are all different. What might the numbers be?</td>
<td>slide 9
discuss these questions with the whole class
slide 10, slide 11</td>
</tr>
</table>

	Give at least two possibilities. Solution: $$\frac{a+b+c+d}{4}=10$$ $$a+b+c+d=40$$ ex. $a=8, b=9, c=10, d=13$. **<u>Exercise 3</u>** We know that $a \geq b \geq c \geq d \geq e = 6$. Find out the median of the following data: a) a, b, c, d, e, b) $2a, 2b, 2c, 2d, 2e$, c) $10a, -10b, 10c, -10d, 10e$. **Solution:** a) c; b) 2c; c) 60.	*slide 12* *slide 13*
Motivation (4 min)	**<u>Human Graph</u>** Discuss the number of siblings that each student has. 10 Ss stand in front of the class. Their task is to represent the number of siblings each S has. *Students with with one sibling sit, students with two siblings kneel, students with three siblings turn, ... and so on.* 	*slide 14* aim: to organize of data
Practice (5 min)	**<u>Exercise 4</u>** The family has 5 members. The mean of the heights is 1.5 m, the modus is 1.2 m and the median is 1.6 m. What is the height of each individual member of the family if you know that in decimeters all of them are integers. ***Solution****: 1.2 m; 1.2 m; 1.6m; 1.7m; 1.8m.*	
Presentation (3 min)	**The Standard Deviation:** The standard deviation of a set of data is a measure of how far the data values are spread out from the mean.	*slide 16* V: **standard**

<table>
<tr><td></td><td>$\delta = \sqrt{\frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + (x_3 - \overline{x})^2 + \cdots + (x_n - \overline{x})^2}{n}}$</td><td>deviation</td></tr>
<tr><td>Practice
(6 min)</td><td><u>Exercise 5</u> For the two sets of data, 4, 5, 6, 7, 8 and 2, 4, 6, 8, 10:
a) find the mean;
b) find the median;
c) find the standard deviation;
d) comment on the similarities and differences in the two data sets.
<table><tr><td>Set of data</td><td>(4,5,6,7,8)</td><td>(2,4,6,8,10)</td></tr><tr><td>mode $\overline{x}$</td><td>6</td><td>6</td></tr><tr><td>median</td><td>6</td><td>6</td></tr><tr><td>standard deviation δ</td><td>$\sqrt{2}$</td><td>$2\sqrt{2}$</td></tr></table>Discussion: The standard deviation for the second data is twice the standard deviation of the first – this is also evident by looking at the two data sets, since the first is spread evenly from 4 to 8 and the second from 2 to 10.</td><td>slides 17-19</td></tr>
<tr><td>Conclusion
(2 min)</td><td>Revision
HW: Find two sets of data, which do not differ in their mode, median but their standard deviations are different.</td><td>slide 20</td></tr>
</table>

Power-point presentation (related to LP 5) – **Introduction to Statistics**

Statistics

Alžbeta Brišová

1

Statistics

- How can we find out about the world around us?

  One way is to collect information, organise it, then study the results and represent them. Collecting and studying information in this way is called statistics.

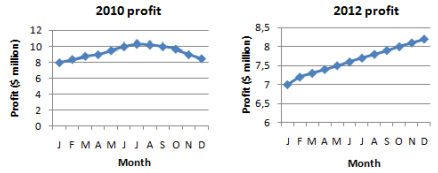
- How can we organize information DATA we have gathered?

  Using tables, graphs,...

2

Newspaper article

- In a newspaper article, the graphs below were accompanied by this text: '**Company profits looked much better in 2012 when compared with 2010**'



What is wrong with these pictures?

3

When we make statements about data that we have collected, we often want to

- say which item is the most popular **mode,**
- which item is in the middle **median,**
- and which item is the average **mean.**

4

Mode, median, mean

What do they tell us?

- They represent different ways of trying to show **the centre of a data set.**
- The type of data affects which you can use.
- The median can only be used for data that is *ordered. It makes no sense to talk* about the median hair colour of the class.

5

MODE

- *mode* is the French word for *'fashion',*
- and it is also the most fashionable (or most popular) value in a set of data;
- there may be more than one mode.

6

MEDIAN

- When a set of values is **arranged** in order of their size, the 'middle value' is the **median**.

2 2 3 3 3 4 5 11 13 18 18 19 21

1 3 4 4 5 7 9 11 13 13 19 21

Median

7

MEAN

- the mean is found by taking the sum of the values, then dividing the result by the number of values;
- the mean is also known as the 'average'.

Find the mean:

2 2 3 3 3 4 5 11 13 18 18 19 21

8

Exercise 1

The canteen at school sells packs with frozen strawberries. This table shows the sales of packs with frozen strawberries for one week.

- Calculate the mode.
- Calculate the mean.
- What is the median?

Day	Frequency
Monday	3
Tuesday	9
Wednesday	3
Thursday	30
Friday	8

9

Answers

Mode: 3; mean: 10.6; median: 8.5.

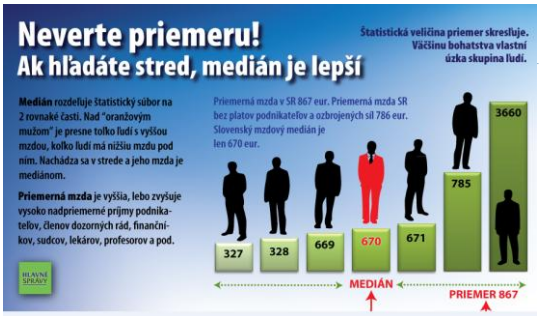
Discuss these questions as a whole class.

- Since these packs takes too much room, the canteen staff wants to know how many pack they should have in the freezer at the beginning of the day. What is your piece advice?

10

- **Would the mode, mean or the median be a good value to rely upon?**
- The **mean** will sometimes give you information that is not very useful.

11



- Source: http://www.hlavnespravy.sk/socialne-polovica-ludi-zarobi-menej-ako-690-eur-mesacne/43980/

12

Exercise 2

- The mean of a set of four numbers is 10. The numbers are all different. What might the numbers be? Give at least two possibilities.

- Solution : $\frac{a+b+c+d}{4} = 10$

$$a+b+c+d = 40$$

ex. $a = 8, b = 9, c = 10, d = 13$

13

Exercise 3

- We know that $a \geq b \geq c \geq d \geq e = 6.$

- Find out the median of the following data:
  a) a, b, c, d, e
  b) $2a, 2b, 2c, 2d, 2e$
  c) $10a, -10b, 10c, -10d, 10e$

Solution
a) c; b) 2c; c) 60

14

HUMAN GRAPH

- Discuss the number of siblings that each student has.

Students

- with one sibling sit, students with two siblings kneel, students with three siblings turn, ... and so

15

Exercise 4

- The family has 5 members. The mean of the heights is 1.5 m, the modus is 1.2 m and the median is 1.6 m. What is the height of each individual member of the family if you know that in decimeters all of them are integers.

- Solution: 1.2 m; 1.2 m; 1.6m; 1.7m; 1.8m.

16

STANDARD DEVIATION

- The **standard deviation** of a set of data is a measure of how far the data values are spread out from the mean.

$$\sigma = \sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+(x_3-\bar{x})^2+\ldots+(x_n-\bar{x})^2}{n}}$$

smerodajná odchýlka

17

Interpreting the standard deviation

- For the two sets of data, 4, 5, 6, 7, 8 and 2, 4, 6, 8, 10:
  a) find the mean;
  b) find the median;
  c) find the standard deviation;
  d) comment on the similarities and differences in the two data sets.

18

Set of data:

Set of data	(4,5,6,7,8)	(2,4,6,8,10)
mode $\overline{x}$	6	6
median	6	6
standard deviation δ	$\sqrt{2}$	$2\sqrt{2}$

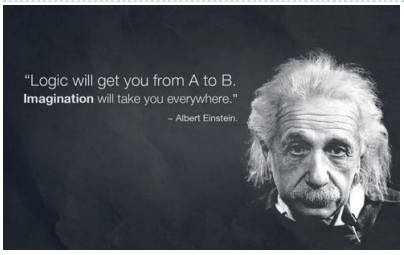
19

SOLUTION c)

- The **mean** of each data set is 6.
- The **median** of each date set is 6.
- The **standard deviation** for the second data is twice the standard devation of the first – this is also evident by looking at the two data sets, since the first is spread evenly from 4 to 8 and the second from 2 to 10.

20

Thank you for your attention



- Source: http://kentz-life.blogspot.sk/2012/04/albert-einstein-quote.html

21

Evaluation of the lesson 5 – **Introduction to Statistics**

Date: 2nd February, 2013 Class: 3rd graders , KGŠM

Level of proficiency in English: A2 – B1 according to CEFR

Lesson plan and Power-point presentation

This lesson bound together the essence of language learning – hearing, seeing and producing language. Presentation provided a support for visual type of learners. They could not only hear but also see the words present in the presentation. This lesson promoted language since students from KGŠM had come across some of the mathematical terms in Slovak language. It was the power-point presentation which was helpful to follow the logical course of the lesson.

My teaching

During my school practice, I have not experienced the students from this class. So the key step was to establish the code of conduct which would enable learners to seize the opportunity to enhance their own learning. Having used experience from the previous teaching and learning via CLIL method, I found it important to acknowledge learners with the tuition. To avoid the negative attitude towards the integration, learners were asked to consider all the possible advantages and contributions of this integration. By this realization, students seemed to feel more relaxed and they were curious about the education. It created a stimulating environment for learning.

Students gave me undivided attention since the course of the lesson was quite unpredictable to them. I think that I achieved the set of objectives.

To keep learners motivated, I paid attention to the clarity of the instructions and the comprehension of learners. However, I think that mathematical content could have been more demanding.

The learners

These learners had positive attitude towards the cooperation. They enjoyed group and pair work. While working in pairs, all students were given an opportunity to produce language. Interest of learners in the integration was shown through many questions they asked.

Lesson plan 6 – **Statistics 2**

Subject: Mathematics **Time**: 45 min

Theme: Correlation Coefficient **Grade**: 3rd graders

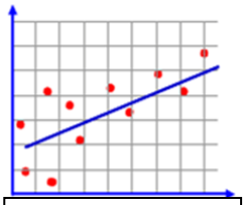
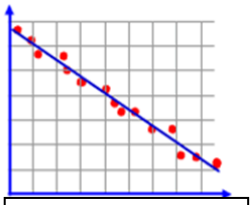
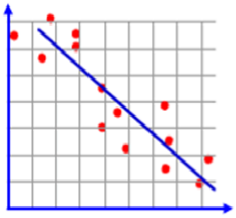
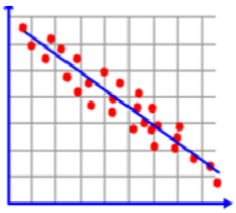
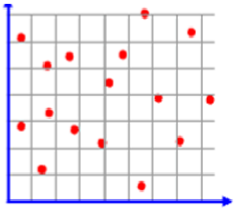
Content objectives	Students will be able to • describe the following terms: *the event, the probability* of occurrence of an event, *complementary event, independent events;* • organize the set of given data, study them and represent the obtained results; • calculate the standard deviation of the given set of data; • decide if two sets of data are related.
Language objectives	Students will be able to • follow instructions provided in English; • describe their solution using words introduced throughout the lesson.
Formative objectives	Students will be able to • cooperate during pair/group work; • tolerate each other during pair/group work.

Language for learning: correlation coefficient, moderate, increase, decrease

Materials: power-point presentation; pieces of papers with graphs and descriptions

Stage & Time	**Procedure**	**Notes**
Introduction (2 min)	Introduction to the lesson (using CLIL method). Ss realize the advantage of this kind of lesson so they can seize the opportunity to learn something new.	*slide 1*
Warm up (4 min)	**Brain Teaser:** Three missionaries and three cannibals want to get to the other side of a river. There is a small boat, which can fit only two. To prevent a tragedy, there can never be more cannibals than missionaries together. How can they get to the other side of the river safely?	*slide 2*

	Solution: 1) 1 cannibal and 1 missionary there, missionary back; 2) 2 cannibals there, 1 cannibal back; 3) 2 missionaries there, 1 missionary and 1 cannibal back; 4) 2 missionaries there, 1 cannibal back; 5) This one cannibal takes the remaining cannibals to the other side.	
Revision (4 min)	How can we find out about the world around us? *One way is to collect information, organise it, then study the results and represent them. Collecting and studying information in this way is called statistics.* How can we organize **data** we have gathered? *(tables, graphs,...)* When we make statements about data that we have collected, we often want to • say which item is the most popular – **mode;** • which item is in the middle – **median;** • and which item is the average – **mean;** What is the measure of how far the data are spread out from the mean? *The* **standard deviation** *of a set of data is a measure of how far the data values are spread out from the mean.*	*slides 3-5* Ss create questions and they answer them as well.
Presentation (5 min)	**Correlation coefficient** **Correlation –** indicates a relationship (connection) between two sets of data. Formula: $r_{x,y} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{s_x . s_y}$ What is the range of the coefficient? *Answer*: $\lvert r \rvert \leq 1$. The closer it is to either -1 or to 1 the stronger the correlation between the two variables is.	*slide 6* *lockstep*

Practice (25 min)	**<u>Exercise 1</u>** Which value of the correlation coefficient r indicates a stronger correlation than 0.72? **a) -0.75** b) 0.65 c) -0.70 d) 0.55 e) 0.60 What is the difference between a negative correlation and a positive correlation? *In a positive correlation as one variable increases, so does the other. In a negative correlation, as one variable increases, the other decreases.* **<u>Exercise 2</u>** Which graphs show strong/moderate/weak correlation? Ss are given 6 pieces of paper with various descriptions of correlations and 6 graphs. Their task is to match graphs with their description. **Solution**:  Strong positive correlation  Weak positive correlation  Strong negative correlation  Weak negative correlation  Moderate negative correlation  No correlation	*slides 7,8* *pair work* *slides 9,10*

<table>
<tr>
<td></td>
<td><u>Exercise 3</u> Find out the correlation coefficient.

<table>
<tr><th>Number of children</th><th>Divorces</th></tr>
<tr><td>0</td><td>4090</td></tr>
<tr><td>1</td><td>4154</td></tr>
<tr><td>2</td><td>2384</td></tr>
<tr><td>3</td><td>372</td></tr>
<tr><td>4</td><td>74</td></tr>
<tr><td>5</td><td>28</td></tr>
</table>
Conclusion
• $r = -0{,}95559$;
• there is strong connection between the number of divorces and the number of dependent children;
• once the number of dependent children increases the divorce rate decreases.</td>
<td>Slide 11
pair work

slide 12
graph</td>
</tr>
<tr>
<td>Conclusion
(5 min)</td>
<td>Ss describe and interpret data they see.

Slide 13 – Ss are given correlation coefficient when comparing these two sets of data. Their task is to interpret data they see.

HW: Choose the theme of the project work. Your task is to study two variables that might be connected. Later you have to collect data, study them and represent results.</td>
<td>slides 14-16</td>
</tr>
</table>

1

Statistics

Alžbeta Brišová

2

Cannibals and Missionaries

Three missionaries and three cannibals want to get to the other side of a river. There is a small boat, which can fit only two. To prevent a tragedy, there can never be more cannibals than missionaries together.

How can they get to the other side of the river safely?

3

Revision

When we make statements about data that we have collected, we often want to

- say which item is the most popular,

  mode,
- which item is in the middle

  median,
- and which item is the average

  mean.

4

Standard Deviation

- The **standard deviation** of a set of data is a measure of how far the data values are spread out from the mean.

$$\sigma = \sqrt{\frac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+(x_3-\bar{x})^2+\ldots+(x_n-\bar{x})^2}{n}}$$

5

Correlation coefficient

- **correlation** – indicates the relationship between two sets of data.
- Formula: $r_{x,y} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{s_x \cdot s_y}$.
- **What is the range of the coefficient?** $|r| \leq 1$
- the closer it is to either -1 or to 1 the stronger the correlation between the two variables is.

6

Correlation coefficient

- Which value of the correlation coefficient *r* indicates a stronger correlation than 0.72?
  a) -0.75
  b) 0.65
  c) -0.70
  d) 0.55
  e) 0.60

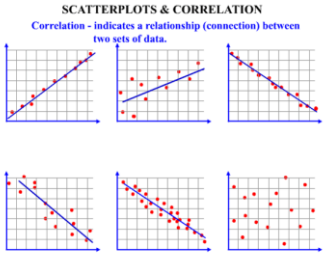
7

What is the difference between a negative correlation and a positive correlation?

- In a positive correlation as one variable increases, so does the other. In a negative correlation, as one variable increases, the other decreases.

8

Which graphs show strong/moderate/weak correlation?



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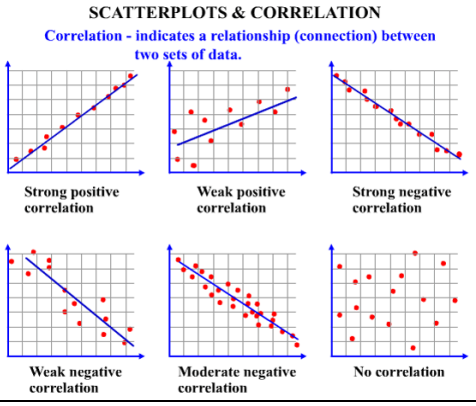


10

Find out the correlation coefficient

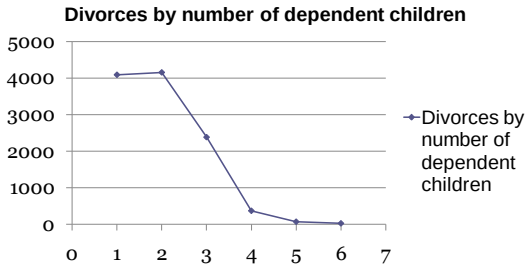
$$r_{x,y} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{s_x \cdot s_y}.$$

Number of children	Divorces
0	4090
1	4154
2	2384
3	372
4	74
5	28

Zdroj: infostat.sk

11

Graph



12

Conclusion:

- r= -0,95559;
- there is strong connection between the number of divorces and the number of dependent children;
- once the number of dependent children increases, the divorce rate decreases.

13

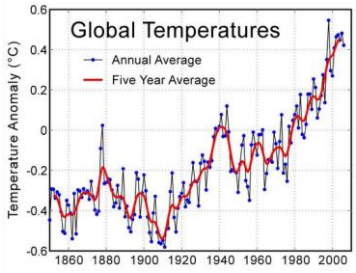
Find out the Correlation Coefficient

Year	1995	1996	1997	1998	1999	2000
Immigrants	3055	2477	2303	2052	2072	2274
Emigrants	213	222	572	746	618	811

Conclusion:
- |r| = 0,789;
- There is a strong relationship between the number of immigrants and emigrants of Slovak Republic.

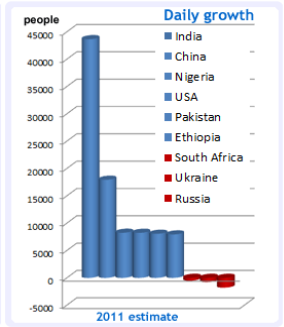
14

Interesting graphs



Source: http://www.newscientist.com

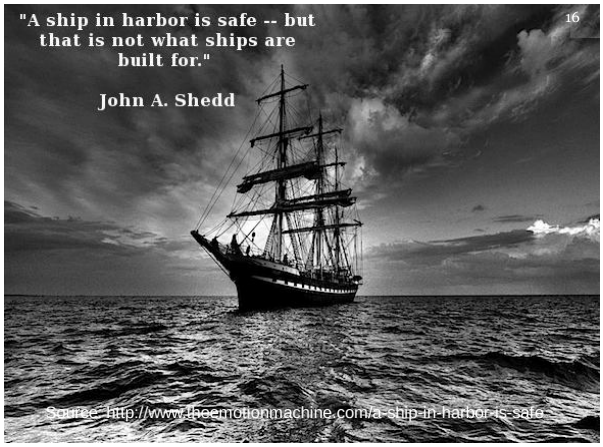
15



Daily growth of population

http://www.geohive.com/

16



Evaluation of the lesson 6 – **Statistics 2**

Date: 11th February, 2013 Class: 3rd graders, KGŠM

Level of proficiency in English: A2 – B1 according to CEFR

Lesson plan and Worksheet

At the beginning of the lesson, I shared the intentions of the lecture with learners to lower their anxiety. Learners could experience the total immersion.

The task input contained stimuli which could enable acquiring of the content and language competences. The mathematical content was more cognitively demanding on the learners than the previous one. It broadened the conceptual understanding of the statistical methods. The importance was given to feedback which should have positive impact on the further motivation to study Mathematics and languages; and it could have an influence on the building the self-esteem.

My teaching

The class split into two groups and one group after the other experienced the lesson. I had realized some mistakes when the first group participated in learning. As the result of an attempt to eliminate these mistakes, the class management was better when I was working with the second groups of students.

Although I could not trace the development of learners for a long time, I tried to give them opportunity to improve their performance in English as well as in Mathematics. Especially the theme of the lesson provided me with authentic tasks for students where they could put theory into practice and acquire the language naturally.

The learners

These learners actively supported each other. So during the lesson students were led to become involved in self- and peer-assessments. As the result of this code of conduct, almost all students were willing to work in pairs. Behaviour of the learners after the warm up activity became spontaneous. I have noticed the incidental language acquisition when students produced language.

2.3 The profile of learners participating in the integration

Under our guidance, altogether 43 students had the opportunity to participate in teaching and learning Mathematics via CLIL. Nineteen students of these attended the second grade of the grammar school Gymnázium Jozefa Gregora Tajovského (abbr. GJGT) and twenty-four students of these attended the third grade of the grammar school Katolícke gymnázium Štefana Moysesa (abbr. KGŠM).

Students attending GJGT are from the class with extended mathematical education. As we have mentioned above, the mathematical content comprised in the lessons corresponds to the standards these students were supposed to reach. Apart from the observation and feedback provided throughout the lessons, these students took a test to determine acquired knowledge and skills in English language as well as in Mathematics.

To find out the level of acquired knowledge of students from KGŠM, we used the method of observation and feedback. These students did not take a didactic test to determine their progress in Mathematics or in English since the focus was on the improvement of speaking skills.

In both classes, the level of English varied from elementary (corresponds to A2 level according to CEFR) to intermediate (corresponds to B1). Most of the students were at *intermediate level*, so they were able to understand language produced by the teacher and they were able to produce grammatically correct sentences. Just a few of the students were at elementary or upper intermediate level.

We had an abiding interest in the approach of students towards lessons using CLIL as the fundamental method in teaching and learning. All students who participated in this kind of education were asked to fill in the questionnaire (see the *Appendix F*).

2.4 The level of acquired knowledge in the learning and teaching Mathematics via CLIL

At this stage we divide students according to the school they attend, since the way of determining acquired skills and knowledge differ.

1) KGŠM, the grammar school

Students from the grammar school KGŠM did not take test to determine their knowledge. It resulted from the linguistic aim of the lessons – the emphasis was put mainly on the developing of speaking skills and production of the language.

By observation and sustained interaction with learners we tried to verify if acquired knowledge and skills by learners were compatible with standards.

Through sustained logical reasoning and argumentation, problem solving, data representation, situation modelling, applying symbolic elements and communication we tried to develop mathematical and language literacy and competencies.

2) GJGT, the grammar school

After the tuition, learners from the grammar school GJGT took the didactic test (see the *Appendix E*). The test was an opportunity for students to demonstrate their knowledge acquired during the CLIL lessons. It consisted of nine tasks; eight of them were obligatory and one of them was a bonus task. The bonus task was introduced to make students feel more comfortable because it was their first taken test which integrated Mathematics and English. Tasks were assigned in English language.

The first five tasks as well as the bonus task were focused on Mathematics. The last three tasks were aimed at reviewing English language. Students could gain maximum 13 points for the content (Mathematics) and up to 10 points for language.

The aims of the test:

- checking the understanding of the mathematical terms: variations with/without repetition, combinations with/without repetition; permutations with/without repetition;
- checking the ability of students to determine the type of task (whether it deals with variations/combinations/permutations) and to use the appropriate formula;
- checking the acquisition of the language for learning;
- checking the ability of students to support their solutions by arguments;
- providing feedback to students as well as for the teacher.

Students were asked to provide logical reasoning and arguments in English language. A great deal of the students (16 out of 19), always provided their reasoning. 3 students just wrote inevitable mathematical terms, no description (except the answer) was included.

These students did not feel confident enough to use English language, although they were able to understand the keyword entry present in the task.

Language and content were graded separately, since the assessment of this test sums up a partial assessment of the subject Mathematics not of the English language. However, to solve the task (to gain some points for content) it was inevitable to understand language for learning introduced throughout the tuition. With this understanding, we found content and language interrelated. The following table shows the results of the test.

	Mathematics (points)	English language (points)
Maximum	13	10
Average	10,79	8,74
Mode	12	10
Median	11	9
The standard deviation	1,78	1,63

Table 1: **The results of the test**

The results of the test show that the majority of students were able to solve the tasks. Since most of the students gained 12 points out of 13 for Mathematics and the median is 11 points, we can say that students were able to use mathematical terms and correctly determined the type of the task (whether it deals with variations, combinations or permutations) and use the appropriate formula to solve it. In parallel with the mathematical content, learners demonstrated their proficiency in the language (necessary for understanding tasks).

Learners manifested the acquired knowledge in language during the lessons as well. They had to become familiar with the language *for* learning to keep up with the pace of the following lessons. The results of the test covering language confirm this fact.

The test examined if mathematical and linguistic literacy of students were developed. On the basis of the fact that students had not been able to solve the tasks found in the test before they participated in the tuition, we subjectively claim that tuition helped learners to improve their competencies in Mathematics as well as in English language.

2.5 The attitude of learners towards the CLIL method in the integration of Mathematics and English language

In our questionnaire, there were thematically four types of inquiries which can be interpreted as follows:

1) *the integration of Mathematics (as a subject) and English language (as a subject);*
2) *facing the challenges and demands related to the CLIL approach;*
3) *advantages and benefits acknowledged by learners;*
4) *the overall approach towards CLIL.*

In the following sections we elaborate these inquiries in deeper details.

1) The integration of Mathematics (as a subject) and English language (as a subject)

Out of 43 learners, only 10 students did not find it appropriate to integrate Mathematics (as a subject) with the foreign language (in our case English language) and 33 learners were supporters of this integration. The supporters would seize the opportunity to participate in the integration of Mathematics and English language again and they thought that they would come across mathematical terminology in the future.

It is noteworthy, that 17 students out of 19 from the class with extended mathematical education appreciated the integration of Mathematics as the content subject with English language. The majority of the students already adopted positive approach towards the subject. They reasoned this conviction that they often come across mathematical terminology in English, and this tuition had lowered their anxiety over English materials.

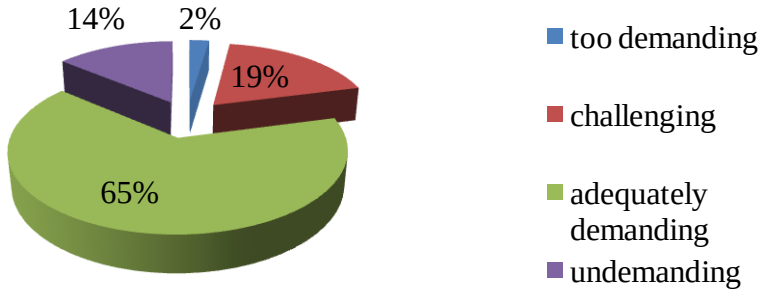
The approach of students from KGŠM to Mathematics varied from the one mentioned above. It was up to the teacher to attract their interest in the subject. 8 students out of 24 did not welcome this integration. They provided arguments that Mathematics is one of the most challenging subjects in general and education in English can make it even more demanding. Although they coped with demands quite well, they were not fond of the tuition.

36 learners would welcome CLIL method in other subjects and they themselves pointed at the demands on the qualification of teachers in the language and in the content subject. They see these demands as the main barrier preventing the employment of CLIL becoming more widespread in the Slovak Republic.

2) Facing the challenges brought by CLIL approach

In general, learners were able to keep up with demands arising from the integration. The following graph shows their personal evaluation of the demands of the tuition.

Graph 1: **Demandingness of the tuition from the point of view of the learners**

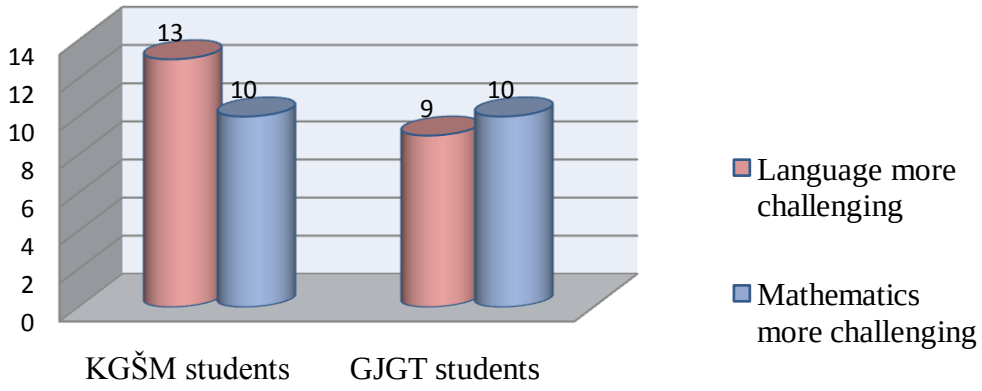


As we can see, 79% of learners found tasks and demands on them compatible with their abilities and skills. 19% of learners saw the tuition as very challenging but manageable. Only 1 student (he/she represents 2% of learners) found the tuition to be incompatible with his/her abilities.

50% of the students believed that the level of mastering the mathematical content acquired by CLIL method could be comparable to the level of mastering if the content had been acquired by the traditional way. The second half of the students believed that foreign language decelerated the absorbing of the content.

Seemingly 23 students promote English language to be more accessible than mathematical content to them. 20 students (which are equally spread between both classes) found content more challenging. The fact that as many as 10 of these students are from the mathematical class can be partly attributable to the relatively demanding theme *Combinatorics*.

Graph 2: **The content versus language challenges**



Since language is the key to understand content, 67% of students saw language barrier as the potential problem for students who did not feel efficient in English. To overcome this problem, students were asked to work in pairs or groups. In conclusion, they admitted that it provoked the cooperation among students.

We found interesting that 18% of the students found it as a disadvantage that they *had to produce the language*. They rationalized this attitude as the result of their long-time low self-esteem in the language. However, especially those students thought that their *self-confidence* in using English language had been built up through *sustained interaction* with language and many opportunities to speak and to listen to the language.

3) Advantages and benefits acknowledged by learners

A great deal of students (39 out of 43) appreciated *genuine and spontaneous improvement* of both – language and content at the same time. They drew their attention to the promotion of *interrelationship* between these subjects.

Many students (90% of the students) found the main contribution of these lessons to their education in the rising *the motivation* towards the English language and Mathematics. *The intercultural task* developed tolerance and cognition of other cultures.

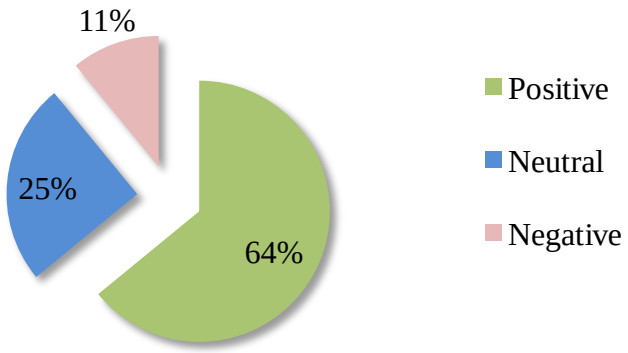
Some of them independently appreciated '*the language in practice*' which they considered absent in the English lessons. They took delight in the deduction of the meaning of the words whose understanding was inevitable in solving tasks. These learners also appreciated '*grammar at work*'. The lessons were attractive to them thanks to innovative methods which CLIL approach requires.

Students became aware of *the immediate practice of language for learning* since they were asked to argument, interpret or reason their solution when solving the tasks, which required the immediate usage of the language occurring in the tasks. All students, regardless their proficiency in English, felt demand on their *conceptual skills* and they realized that it made them *to think in English language*. They claimed it to be one of the most persuasive evidence of the value of the CLIL.

4) The overall approach towards CLIL.

In conclusion, 64% of learners had positive attitude towards this approach, 25% had neutral attitude and 11% preferred traditional teaching and learning, as it is shown in the graph 3.

Graph 3: **The overall aproach towards CLIL**



Students saw the necessity of being proficient in English language. Learners also appreciated innovative methods and tasks used during the tuition which had impact on their conceptual thinking and personality. They recognized the idea of the integration and its advantages and challenges. As we have mentioned above, 84% of students would welcome at least one more experience with the CLIL approach.

CONCLUSION

In recent years, the Content and Language Integrated Learning with its acronym CLIL have become fast spreading across the Europe. Educational authorities have recognized underlying values of this phenomenon and have recommended its integration to become a part of the mainstream education in the European countries. The most outstanding among its benefits is the preparation of learners for the life in multicultural society by significantly improving language competences and by deepening tolerance and cultural awareness of the mother country, target countries and also of other countries.

Slovak schools have offered CLIL provision in foreign as well as in regional and minority languages for decades. Slovakia has had long-term experience with the CLIL phenomenon but the catalysts for broadening the provision of this method were the European programmes in education and training. Although, this method is not new, it still has to take on many challenges, such as the lack of trained teachers and shortage of materials appropriate for CLIL method.

Aware of the challenge facing lack of materials, we have taken an interest in designing and testing own materials and it became the principle aim of our diploma thesis. The objective of this thesis, which deals with the integration of Mathematics and English language, was to design and verify teaching and learning materials in school practice concerning the Combinatorics, Probability and Statistics theme at grammar schools.

The created CLIL materials inhered in the practical part consist of the detailed lesson plans and their related worksheets or power-point presentations. This materials sum up the main contribution of this work. Lesson plans and their related worksheets were tested in school practice. Two classes of different grammar schools participated in the teaching and learning Mathematics via CLIL. One group of students were from Gymnázium Jozefa Gregora Tajovského (GJGT). These second graders attended class with extended mathematics education. The other group of students were third graders at Katolícke gymnázium Štefana Moysesa (KGŠM).

Students acknowledged this type of tuition so they were better prepared to take opportunity to improve in both mathematical content as well as in English language. Knowledge which they acquired was monitored through feedback during the lessons. Moreover, students from the GJGT took a test, which provided us with valuable

information about the acquired knowledge in Mathematics and in English. We found out that knowledge and skills gained during the tuition were correspondent to the educational standards.

Of our interest was also attitude of learners towards this type of educational provision. Via questionnaire, we found out approach of learners and other information about the CLIL method from their point of view.

Obtained data have proved very encouraging. Students saw the necessity of being proficient in English language and believed that CLIL could ensure them to be more receptive to multilingualism. In general, they took the opportunity to enhance not only their education but also their personality development through building self-confidence when using the target language.

Learners also appreciated innovative methods and tasks used during the tuition which had impact on their conceptual thinking and personality. They recognized the idea of the integration and its advantages and challenges. Among these recognized advantages of CLIL was rising the motivation towards learning languages and other cultures. Other advantages concern the immediate practice of language for learning. Students were asked to explain or interpret their solutions while solving the tasks, which required the immediate usage of the language occurring in the task. In this kind of tasks there was also a demand on their conceptual thinking.

In conclusion, the designed materials are suitable for teaching and learning the Combinatorics, Probability and Statistics theme at grammar schools. They provide the teacher with mathematical and English content which can be grasped by their students.

Lesson plans, worksheets and presentations can be used by teachers of Mathematics who want to use the CLIL method to enhance education of learners.

RESUMÉ

„Teória ostane obyčajnou teóriou, kým neprikročíme k činu.“

J.A. Komenský

Viacjazyčnosť sa stáva významnou zložkou identity občana Európy. Výchovné a vzdelávacie autority rozpoznali prínos metódy CLIL (obsahovo a jazykovo integrované vyučovanie) v napomáhaní obyvateľom Európy a v adaptácii na život v multikultúrnej spoločnosti. Vďaka svojím výhodám sa tento inovatívny prístup k vyučovaniu a učeniu sa stáva stále viac rozšíreným a je oň nezvyčajný záujem.

Hlavná črta tohto fenoménu spočíva v integrácii odborného predmetu s cudzím, s regionálnym alebo s menšinovým jazykom. Z dlhodobého hľadiska kladie CLIL stabilný apel na nadobúdanie zručností a rozvíjanie kompetencií v obidvoch predmetoch.

CLIL je vyučovacia metóda, to znamená, že v sebe zahŕňa taký postup činností učiteľa a žiaka, ktorý vedie k dosiahnutiu výchovno-vzdelávacích cieľov. Ideálom tejto metódy je ponúknuť jazykovo rozšírené vyučovanie, ktoré vedie k funkčnej dvojjazyčnosti študentov, t.j. študent má kompetencie v dvoch jazykoch (rodný a cieľový) v určitej oblasti odborného predmetu.

CLIL vytvára optimálne a prirodzené podmienky pre študentov na rozvoj myslenia v cudzom jazyku a v štyroch základných zručnostiach v jazyku: v čítaní a počúvaní (receptívne zručnosti) a v rozprávaní a písaní (produktívne zručnosti). Jedným z najväčších pozitív metódy CLIL je aj to, že núti študentov rozmýšľať v jazyku.

Benefitom metódy je, že od začiatku výučby je poskytovaných množstvo podnetov determinujúcich kognitívny rozvoj študentov. Keďže koncepty a pojmy týkajúce sa matematického obsahu sú prezentované i osvojované v anglickom jazyku, myslenie žiakov smeruje k sofistikovanejšej úrovni.

Stálou interakciou s jazykom sa očakáva, že u učiacich sa dôjde k rozvinutiu kompetencií v anglickom jazyku, a to by mohlo viesť k zvýšeniu ich sebavedomia a vlastného ocenenia. Takto je stimulovaný pozitívny prístup študentov k učeniu sa nielen jazykov ale aj matematiky.

Európske programy týkajúce sa edukácie študentov i učiteľov poslúžili ako katalyzátor na rozsiahlejšie zavádzanie tejto metódy na Slovensku. Aj keď má slovenské školstvo dlhoročné skúsenosti s týmto prístupom k učeniu, jeho implementácia ešte stále čelí problémom, spomedzi ktorých dominuje nedostatok učiteľov kompetentných vyučovať touto metódou či deficit materiálov vhodných na výučbu metódou CLIL. Budúci

rozvoj tejto metódy závisí aj od realizovania formálnej prípravy učiteľov a ich ďalšej kvalifikácie.

Uvedené problémy spojené so zavádzaním tejto metódy boli jedným z primárnych determinantov realizácie predkladanej práce. Ďalším podnetom bol náš záujem o inovatívne metódy, ktoré majú vplyv na celostný rozvoj osobnosti študenta. Metóda CLIL k nim nepochybne patrí.

V tejto našej diplomovej práci sme CLIL využili ako nástroj na vyučovanie a učenie sa tematického okruhu *Kombinatorika, pravdepodobnosť a štatistika* na stredných školách. Cieľovým jazykom je anglický jazyk, ktorý sa v súčasnej dobe stáva nevyhnutným fundamentom pre osobný a kariérny rast a pre mobilitu občanov nielen v rámci Európy.

Práca je rozdelená na teoretickú a praktickú časť. Teoretická časť pozostáva zo šiestich kapitol. Prvé tri kapitoly ponúkajú pohľad na koncepty, ciele a princípy úzko súvisiace s týmto typom integrácie. Uvedené sú aj základné smerodajné princípy, ktorých dodržiavanie je potrebné, aby sa CLIL stal účinným. Zaraďujeme k nim obsah, komunikáciu, poznanie a kultúru.

Prechod od teórie k praxi vyžaduje od učiteľa implementovanie pomerne náročného komunikačného princípu, ktorý zahŕňa tri súvisiace typy jazyka používané na rôzne účely – *jazyk na učenie sa* (nevyhnutný k osvojeniu obsahu), kľúčovým je *jazyk pre učenie* (potrebný pre prácu a realizáciu sa v cudzojazyčnom prostredí) a *jazyk cez učenie*, ktorý sa nedá naplánovať, ale vyskytuje sa v priebehu hodiny. Komunikačný princíp zaisťuje nielen uvedomelý pokrok študentov v jazyku, ale hlavne posledné dva uvedené typy jazyka smerujú k neuvedomelému osvojovaniu kompetencií, vyplývajúcemu z nepretržitej interakcie študenta s cudzím jazykom. Za kľúčový faktor, ktorý vplýva na úroveň rozvoja kompetencií a formovanie postojov študentov k integrácii, považujeme uvedomenie si výhod sprostredkovaných cez metódu CLIL.

Štvrtá kapitola práce predostiera spomínané výhody a prínos metódy nielen pre učiaceho sa, ale aj pre vyučujúceho. Okrem vyššie spomínaných výhod chceme upriamiť pozornosť čitateľa aj na ponúknutý priestor na interpersonálnu komunikáciu a rozvoj kooperácie. Táto metóda nie je selektívna, to znamená, že študenti participujúci na vyučovaní a učení sa nie sú vybratí na základe jazykových schopností. CLIL prispieva ku kvalite vyučovacieho aj učebného procesu, na ktorú má každý študent nárok.

Zavedením metódy CLIL na Slovensku sa zaoberá piata kapitola teoretickej časti. Ponúknutý je pohľad na štatút jazykov v integrácii v slovenských školách. Zdôrazňujeme, že najmä stredné odborné školy a učilištia si uvedomujú potrebu a potenciálny prínos

zavedenia metódy CLIL pre ich študentov, ktoré by im umožnili mobilitu za hranicami nášho štátu.

Uvádzame požiadavky na matematickú gramotnosť a kompetencie žiaka, ktorých získanie sa viaže k tematickému celku, na ktorý je zameraná naša pozornosť. Od študentov gymnázia sa v rámci celku *Kombinatorika, pravdepodobnosť a štatistika* vyžaduje logické myslenie a argumentácia, riešenie problémov, reprezentácia dát, modelovanie situácií a mnohé iné. Metóda CLIL vedie žiakov k uvažovaniu, diskusii a vyžaduje ponor do problému. Vedie k dosiahnutiu najvyššieho stupňa matematických kompetencií podľa PISA – k úrovni reflexie.

Jazyková edukácia na Slovensku korešponduje so *Spoločným európskym referenčným rámcom pre jazyky (CEFR).* Vytýčeným cieľom edukácie je umožniť cez jazykové vzdelávanie študentom gymnázií dosiahnuť úroveň B2. V prípade realizácie tejto práce, vyučovanie metódou CLIL ponúklo študentom priestor na jazykové vzdelávanie bez zvýšenia celkového počtu hodín pre predmet Anglický jazyk zo strany školy.

Kritici opisovanej metódy často poukazujú na potláčanie materinského jazyka. Ako sme už skôr uviedli, ideálom tohto prístupu je funkčný bilingvalizmus, to znamená, že študenti disponujú matematickou terminológiou aj v materinskom jazyku. Domnievame sa, že v niektorých prípadoch vedie vyučovanie v cudzom jazyku k hlbšiemu uvedomeniu si vlastnej kultúry a rozmanitosti materinského jazyka.

Praktická časť diplomovej práce predstavuje náš najväčší prínos – obsahuje návrhy a overovanie CLIL materiálov v školskej praxi. Je rozdelená na päť kapitol. V prvej kapitole analyzujeme návrhy týchto materiálov a špecifiká ich tvorby. Do pozornosti kladieme nároky na učiteľov. Zdôrazňujeme taktiež vytýčenie si cieľov hodiny, ktoré sme rozdelili do troch partikulárnych oblastí: ciele súvisiace s matematickým obsahom, ciele súvisiace s osvojovaním si jazykových kompetencií a tretiu skupinu predstavujú formatívne ciele, t.j. ciele súvisiace s rozvojom osobnosti žiaka. Charakteristickou črtou všetkých príprav je uvedenie *jazyka na učenie sa,* vyššie spomínaného medzi typmi jazyka, ktorý slúži študentom na postupné osvojovanie si vedomostí a zručností. Pre učiteľa poskytuje oporu v štruktúrovaní pripravovaných hodín. Učiteľ využitím vhodných stratégií a metód napomáha žiakovi v jeho procese učenia sa.

V druhej kapitole praktickej časti sa nachádzajú vytvorené materiály, ktoré zahŕňajú podrobné plány vyučovacích hodín a k nim prislúchajúce pracovné listy či power-pointové prezentácie v rámci tematického okruhu *Kombinatorika, pravdepodobnosť a štatistika*.

Štyri takto vytvorené vyučovacie a učebné materiály, ktoré zahŕňajú kombinatoriku, boli overené v Gymnáziu Jozefa Gregora Tajovského (ďalej GJGT) a dva z nich, týkajúce sa štatistiky, v Katolíckom Gymnáziu Štefana Moysesa (ďalej KGŠM). V prílohách sa nachádzajú neotestované materiály k téme pravdepodobnosť.

Ku každej odučenej hodine uvádzame aj subjektívne hodnotenie, ktoré pozostáva z troch častí: hodnotenie materiálu (zväčša zhŕňame mieru využitia príprav a súvisiacich pracovných listov či power-pointových prezentácií), hodnotenie realizácie vyučovacieho procesu (vyjadrujeme vlastný postoj k miere naplnenia stanovených cieľov, k najzaujímavejšej a najmenej zaujímavej časti hodiny, k náročnosti jednotlivých častí) a hodnotenie žiakov (najčastejšie uvádzame úlohy, ktoré viedli k motivácii študentov, taktiež tie, ktoré spôsobovali učiacim sa problémy, prípadne popisujeme atmosféru na hodinách).

Takmer všetky odučené vyučovacie hodiny majú spoločnú úvodnú aktivitu – matematické hádanky, ktorým prikladáme dôležitý význam nielen kvôli motivačnému prínosu. Okrem toho, že pomáhajú vytvárať priateľskú atmosféru, sú vďaka nim od začiatku hodiny kladené požiadavky na kognitívne myslenie študentov. Očakáva sa, že učiaci sa opíšu riešenia v cieľovom jazyku, t.j. tieto hravé aktivity zabezpečujú prvotný kontakt s jazykom.

Vo vytvorených prípravách sme uviedli vlastné cvičenia a úlohy a rovnako aj úlohy z iných materiálov. Každá úloha obsahuje jazykové a kognitívne požiadavky na študenta, ale ich úroveň sa líši. Niektoré úlohy boli náročnejšie po kognitívnej stránke, iné kládli dôraz na rozvoj jazyka. Z motivačného hľadiska sme považovali za dôležité, aby úlohy a nároky kladené na žiakov korešpondovali s ich kompetenciami v matematike aj v anglickom jazyku, avšak aby zároveň poskytovali možnosť pokroku v odbornom predmete aj v cieľovom jazyku. Jednotlivé časti materiálov sú uvedené v jazyku, v akom boli odučené.

Študenti už počas hodín dostávali aj dávali spätnú väzbu – ich záujem či prípadnú nezainteresovanosť alebo porozumenie inštrukciám. Monitorovaním sme získali informácie, ktorými bol ovplyvnený ďalší priebeh hodiny.

Profil študentov zúčastnených na edukácii metódou CLIL sa nachádza v tretej kapitole praktickej časti. Študenti druhého ročníka GJGT sú z matematickej triedy – požiadavky na hĺbku a rozsah osvojenia matematického obsahu sú rozsiahlejšie ako na študentov z klasickej triedy. Navrhnuté materiály, ktoré boli v tejto triede otestované odrážajú uvedené nároky. Študenti GJGT mali pozitívny prístup k matematike

ako odbornému predmetu, čím sa líšili od skupiny žiakov tretieho ročníka v KGŠM, kde bol učiteľ v role neustáleho motivátora k štúdiu matematiky.

Jazykové zručnosti a kompetencie študentov sa v obidvoch triedach pohybovali v rozmedzí úrovní od A2 – B1, v klasifikácii podľa CEFR. Väčšina študentov bola schopná porozumieť môjmu jazykovému prejavu a aj samostatne sa vyjadrovať v anglickom jazyku. Iba v ojedinelých prípadoch žiaci nerozumeli vstupným informáciám v úlohách, avšak práve táto rozdielnosť úrovní vytvorila priestor na kooperáciu študentov a rozvoj interpersonálnych kompetencií. Títo študenti ocenili podporný materiál *Language for learning* (Jazyk na učenie sa), ktorý im pomohol zlepšiť sa v jazykovom prejave.

V štvrtej kapitole podávame pohľad na získané vedomosti a zručnosti cez integráciu. U študentov z KGŠM sme ich osvojenie priebežne monitorovali pozorovaním. Požadovali sme od nich logické odôvodnenie riešení, reprezentáciu dát, modelovanie situácií, používanie matematických symbolov a komunikáciu, čím sme sa snažili o rozvoj matematických a jazykových kompetencií. Väčšina žiakov bola schopná zaradiť do svojho ústneho či písomného prejavu *jazyk na učenie sa*.

Preferovanou organizačnou formou učenia sa v obidvoch triedach bola práca v dvojiciach alebo v menších skupinách, čím boli utvorené podmienky na rozvíjanie produktívnych zručností a aj menej sebaistí žiaci mohli komunikovať v cieľovom jazyku.

Študenti z GJGT mali príležitosť preukázať nadobudnuté vedomosti v didaktickom teste (*Appendix E*), ktorý overoval ich zručnosti v kombinatorike i v anglickom jazyku. Výsledky testu poskytovali spätnú väzbu ako pre učiteľa tak pre žiakov. Úlohy testu boli zadané výlučne v cieľovom jazyku. Študenti pracovali samostatne.

Informácie získané takýmto spôsobom subjektívne hodnotíme ako veľmi potešujúce. Z 19 študentov zúčastnených na výučbe metódou CLIL, 16 pri riešení testových úloh argumentovalo riešenia v anglickom jazyku používajúc *jazyk na učenie sa* uvedený počas edukácie. 3 študenti zdôvodnili riešenia v slovenskom jazyku, avšak odpovede uviedli v anglickom jazyku. Na základe správneho riešenia úloh usudzujeme, že aj títo študenti boli schopní samostatne porozumieť kľúčovým slovám a vzťahom v zadaní.

Jazyková a matematická stránka boli hodnotené samostatne, keďže výsledky testu sa započítavali do hodnotenia z predmetu matematika a nie z predmetu anglický jazyk. Napriek oddelenému hodnoteniu upozorňujeme, že východiskom správneho vyriešenia úlohy – k získaniu bodov za obsah – bolo pochopenie zadania v cieľovom jazyku.

Podľa dosiahnutých výsledkov v teste sa zdá, že študenti dosiahli veľmi dobrú úroveň osvojenia si matematických pojmov a vzťahov medzi nimi. Jazykovú zdatnosť

študenti predviedli už počas vyučovacieho procesu. Výsledky testu naznačujú naplnenie jazykových cieľov jednotlivých hodín.

Piata kapitola teoretickej časti popisuje postoj študentov k integrácii, zisťovali sme ho dotazníkom (pozri *Appendix F*), skúma štyri zložky prístupu učiacich sa, ktoré môžeme popísať ako *postoj k integrácii matematiky (ako odborného predmetu) a anglického jazyka (ako cieľového predmetu), postoj k výzvam a nárokom inovatívneho prístupu CLIL k učeniu a učeniu sa, výhody metódy CLIL uznané študentmi účastnými na integrácii, celkový prístup študentov k metóde CLIL.*

33 študentov uvítalo integráciu dvoch uvedených predmetov a myslia si, že sa s matematickou terminológiou v anglickom jazyku stretnú aj v budúcnosti. Prínos tejto výučby vidia aj v jej vplyve na formovanie pozitívneho prístupu k cudzojazyčnej študijnej literatúre.

10 študenti, z toho len 2 z matematickej triedy, nepovažovali dané predmety za vhodné na integráciu. 8 študenti z KGŠM, ktorí zastávali negatívny postoj, pokladali obsah matematiky za náročný v materinskom jazyku a táto náročnosť sa podľa nich zvyšuje s prezentáciou a osvojovaním si obsahu v cudzom jazyku. Aj napriek tomu sme sa snažili motivovať študentov a vzbudiť v nich záujem o vyučovanie a učenie sa metódou CLIL.

36 študentov z celkového počtu 43 zúčastnených na výučbe by uvítalo aj integráciu iných predmetov s cudzím jazykom. Uvedomujú si však aj nároky na kvalifikáciu učiteľa a jeho kompetencie a vidia to ako prekážku rozšírenejšieho zavádzania tejto metódy.

Druhou sledovanou zložkou prístupu študentov bol ich postoj k náročnosti a k predkladaným výzvam. Podľa výkonov preukázaných na hodine usudzujeme, že väčšina študentov bola schopná zvládať na nich kladené požiadavky. Čo sa týka pohľadu študentov, 79% z nich si myslí, že nároky súvisiace s metódou CLIL sú v súlade s ich kompetenciami. 19% študentov sa zdala edukácia primerane náročná a zvládnuteľná. Len 1 študent/študentka (2%) nepovažoval(a) nároky tejto výučby v súlade s jeho/jej kompetenciami.

50% študentov si myslí, že miera osvojenia matematického obsahu prostredníctvom anglického jazyka by korešpondovala úrovni osvojenia si obsahu prostredníctvom materinského jazyka. Ostatní študenti usúdili, že anglický jazyk znižoval rozsah (nie však úroveň) matematického obsahu, s ktorým sa stretli počas hodín.

20 učiaci sa, ktorí sú rovnomerne rozmiestnení medzi obe skupiny študentov, považovali matematiku za menej prístupnú v porovnaní s obsahom predmetu. Zvyšným študentom sa zdal náročnejší jazyk. To, že až 10 študentom z matematickej triedy sa zdal náročnejší obsah pripisujeme náročnosti kombinatorickej témy a tomu, že mnohí z týchto študentov sa na škále úrovní v anglickom jazyku približovali k úrovni B2 a teda neuvedomujúc si jazykové výzvy, upriamili svoju pozornosť na matematiku. Na základe získaných informácií usudzujeme, že prevažná väčšina študentov si uvedomovala jazykovú stránku a bola znížená ich sústredenosť na obsah odborného predmetu. Z našich doterajších skúseností vyplýva, že keby mali študenti možnosť participovať na dlhodobejšej edukácii metódou CLIL došlo by k zautomatizovaniu jazykových činností a k výraznejšiemu spontánnemu prejavu v anglickom jazyku.

Kľúčovým faktorom k osvojeniu si obsahu predmetu je jazyk a to mnohí študenti vidia ako nevýhodu metódy CLIL pre tých, ktorých jazykové kompetencie neumožňujú pochopenie obsahu. Neporozumeniu obsahu odborného predmetu zo strany študentov sme sa snažili predchádzať prácou vo dvojiciach alebo v skupinách. Takto sa vytvoril priestor na spoluprácu a pomoc menej zdatnejším študentom, či už priamym prekladom alebo opisom problému. Uľahčilo to aj monitorovanie pochopenia učiva jazykovo zdatnejšími študentmi.

Považujeme za zaujímavé, že až 18% študentov vidí ako nevýhodu tejto metódy to, že sa od nich požaduje ústny a písomný prejav, čiže produkcia jazyka. Študenti obhajovali uvedený názor tým, že nie sú dostatočne sebavedomí, aby prezentovali jazykové zručnosti a kompetencie. Snažili sme sa im vysvetliť, že hlavným cieľom hodín, ktoré využívajú metódu CLIL, je komunikácia – vzájomná výmena informácií a aj chyby v prejave sú významnou formou učenia sa. Väčší význam sa pripisuje plynulosti a schopnosti vyjadriť svoje myšlienky ako na gramatickú korektnosť vyjadrovania sa. Väčšina takýchto študentov v závere uviedla, že cez stálu interakciu s jazykom a vďaka mnohým príležitostiam prijímať a produkovať jazyk, postupne dochádzalo k budovaniu ich sebavedomia. Niektoré zručnosti a schopnosti v jazyku mohli nadobudnúť cez písomné vypracovanie úloh uvedených v pracovnom liste.

Tretia zložka sa týka výhod, ktoré si študenti uvedomovali počas vyučovacieho procesu a mohli tak využiť príležitosť k obohateniu. 39 študentov ocenilo duálny cieľ tejto metódy – paralelné zlepšenie sa v matematike aj v anglickom jazyku. Značným pokrokom boli motivovaní k záujmu o štúdium obidvoch predmetov.

Nezávisle od seba poukázali mnohí študenti na autentickosť jazyka a úloh, ktoré považujú za absentujúce na hodinách anglického jazyka. Uvedomili si okamžité precvičovanie novej slovnej zásoby, keďže sa vyskytovala prevažne v zadaní úloh a pri popise riešenia boli nútení manipulovať ňou. Študenti tak mohli získať pragmatické vedomosti a zručnosti, ktoré podporovali ich spontánnosť v používaní anglického jazyka. Mimoriadny záujem vyvolala prezentovaná gramatika, ktorú mali študenti možnosť precvičiť si v praxi. So záujmom sa snažili dedukovať význam nových slov.

Využitie metódy CLIL vo vyučovaní vyžadovalo aj implementovanie ďalších inovatívnych metód, ktoré sa často používajú v zážitkovom učení. Tie dali edukácii charakter, ktorý bol pre študentov nový a motivujúci. Dodržiavanie princípov metódy CLIL, ako i zavádzanie inovatívnych metód môže viesť k lepšej úrovni vyučovania.

Všetci študenti, bez ohľadu na ich zdatnosť v cieľovom jazyku, pociťovali nároky na kognitívne myslenie, na to *akým spôsobom myslia*. Zvýšila sa kvalita učenia sa cez lepšie asociácie konceptov. Uvedomenie si skutočnosti, že už po niekoľkých vyučovacích hodinách myslia v anglickom jazyku, bolo pre študentov najpresvedčivejším dôkazom o pozitívnom vplyve metódy CLIL.

Posledná zložka postoja zastrešuje ostatné. Po zvážení výhod a nevýhod tejto metódy 74% študentov, ktorí participovali na integrovanom vyučovaní a učení sa pod naším vedením, uviedlo pozitívny postoj k metóde CLIL. 12% malo neutrálny vzťah a 14% preferuje tradičnú formu edukácie.

Študenti si uvedomujú nevyhnutnosť ovládania anglického jazyka pre ich ďalší osobný a kariérny rozvoj. Uviedli, že skúsenosťou s integráciou bola zvýšená ich motivácia k štúdiu jazykov aj matematiky. Ako sme spomínali, uvítali by ďalšiu skúsenosť s metódou CLIL.

CLIL je jednou z tých inovatívnych metód, ktoré kladú dôraz na komplexný rozvoj osobnosti študenta a vyžadujú rozvoj učiteľa po jeho odbornej aj osobnostnej stránke. Ako povedal Gabriel Laub „Pokrok? Áno - po kroku, po kroku...“ – možno aj táto práca je malým krokom k pokroku študentov, ktorí participovali na výučbe touto metódou. Nami vytvorené materiály môžu poslúžiť aj iným učiteľom matematiky, ktorí sa rozhodnú použiť metódu CLIL ako nástroj pre edukáciu.

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THE LIST OF APPENDICES

Appendix A

Lesson plan 7 – **Introduction to Probability**

Subject: Mathematics **Time**: 90 min

Theme: Probability **Grade**: 3rd graders

Content objectives	Students will be able to • describe the following terms: *the event*, *the probability* of occurrence of an event, *complementary event*, *independent events;* • give own examples of certain events, possible but not certain events and impossible events; • cite through examples that probability of occurrence of an event is a non-negative fraction, not greater than one; • use permutations and combinations in solving problems in probability; • use the formula for determining the probability of the occurrence of an event in the exercises similar to those presented in the lesson; • state the multiplication theorem on probability for independent events.
Language objectives	Students will be able to • use new vocabulary (such as **the probability of (doing), more, equally, less likely**) in their speech; • comprehend and retell problems; • produce sentences in present simple tense.
Formative objectives	Students will be able to • tolerate each other during pair/group work; • cooperate during pair/group work.

Language for learning: the probability of (doing) (SYN likelihood, chance), high probability (a strong probability), low probability, probable (likely to happen), probably (adverb), more likely, equally likely, less likely, favourable (BrE), favorable (AmE), coin flipping, complementary probability, Gambler´s Fallacy

Materials: worksheets, dice

Stage & Time	Procedure	Notes
Introduction Warm up (10 min)	Introduction to the lesson (using methodology CLIL). Topic: PROBABILITY. **STEP 1** Each couple is given a die and throws it 20 times. Couples write down the total figure of the **event** 'obtaining 6'. One student writes the sum of all the figures on the blackboard. (Let *F* be the sum). An **event** is one or more outcomes of an experiment (an event is the subset of all outcomes that can occur). Notation for the events: *A, B,...* How many throws were there altogether? *N – the number of students. Altogether there were* $\frac{N}{2} \cdot 20$ *throws.* 'Number 6 landed *F* times out of $N \cdot 10$ throws.' Express it as a fraction. $\frac{F}{10 \cdot N}$. **STEP 2 Exercise 1 a)** How many possible outcomes does this experiment have? *There are six possible outcomes:* 1,2,3,4,5,6. Is there any outcome **more likely** to occur than the others? *No, they are* ***equally likely*** *to occur. A fair die – it has an equal chance of landing on any of its sides.* How many ***favourable*** outcomes does the experiment have? Favourable, in our case, means 'obtaining 6'. Favourable = all the outcomes that meet my conditions. *Just one outcome: 'obtaining 6'.* Express as a fraction, as a decimal, as a percentage: One favourable outcome out of six possible outcomes. As a fraction: $\frac{1}{6}$; as a decimal: 0,166; as a percentage:	WS(s) *pair work* V: **event** one S at the blackboard V: **more likely, equally likely** **favourable** (BrE)/ **favorable** (AmE)

	16,6%. **STEP 3** Compare figures obtained in STEP 1 and STEP 2. In STEP 1 we obtained *empirical probability* (based on an experiment) and in STEP 2 we obtained *theoretical probability* of obtaining 6. As an experiment is repeated more and more times, the proportion of outcomes favourable to any particular event will tend to come closer and closer to the theoretical probability of that event. In other words, if you do a large number of trials you will get a more accurate result. **Exercise 1 b)** How many possible outcomes are there? *There are six possible outcomes: 1,2,3,4,5,6.* How many favourable outcomes are there? *There are three possibilities: 2, 4 and 6.* What is the probability of obtaining an even number? *The probability of obtaining an even number is* $\frac{3}{6}$. *It is the same as ½ or 0.5 or 50%.* Now, using the knowledge from the previous examples, can you determine the probability of an event?	
Presentation (10 min)	Use your own words to describe the term 'the probability of the event'. **Probability** is the measure of how likely an event is. Notation for the probability of the event A is *P(A).* $P(A) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$. ***Pravdepodobnosť*** je hodnota vyčísľujúca istotu resp. neistotu výskytu určitej *udalosti*. How do we express probability? *Mathematics uses*	*lockstep*

	numbers to describe probabilities. Probabilities can be written as fractions, decimals or percentages. You can also use a probability scale, starting at 0 (impossible) and ending at 1 (certain). (It means that $0 \leq P(A) \leq 1$.) Convert a fraction to a percentage: $\frac{3}{5}$. (Answer: 60%.) Some events are *impossible*, other events are *certain to occur*, while many are *possible, but not certain* to occur. **Language for learning:** **Exercise 2)** Give me same examples of • **impossible** events: *the sun turning to ice; Shrek becoming President of the U.S.; finding the purple colour on the American flag...* (nemožný); • **certain** events: *the number 10 is less than the number 11; George Washington was the first U.S. President; flipping a coin and getting either a head or a tail ....* (istý jav); • **possible**, but not certain events: *picking the King of Hearts from a deck of cards; rolling a 3 on a single die* (možný). What is the probability of the event that is certain to occur? Use the scale 0-1. *The probability of the event that is certain to occur is 1. P(A)=1.* What is the probability of the event that is impossible to occur? Use the scale 0-1. *The probability of the event that is impossible to occur is 0. P(A)=0.* What is the probability of the event that is possible to occur? Use the scale 0-1. *The probability of the event that is possible is ≥0 and ≤1. It says that 0≤P(A) ≤1.*	Who or what is Shrek? What tense do you use to describe general truths, facts and scientific laws? *Present simple tense.*

	Náhodný jav – jav s pravdepodobnosťou P, pričom $0 \leq P \leq 1$.	
Practice (7 min)	What is wrong with the following statement? 'The probability of obtaining a 6 when I throw a die is $^1/_6$ – so if I throw the die six times I should get exactly once 6.' *In theory this statement is true, but in practise it might not be the case. Try throwing a die 6 times - you won't always get one 6. In other words, if you do a large number of trials you will get a more accurate result.* When is it helpful to know the probability of the event? *Chemistry, Physics, Lottery, and Medicine (to find out if the medicine is helpful)...* What is **coin flipping** or **coin tossing**? *Coin flipping, coin tossing, or* ***heads or tails*** *is the practice of throwing a coin in the air to choose between two alternatives, sometimes to resolve a dispute between two parties.* Tell me any event when we flip a coin for this reason. (*Coin flipping is used to decide which end of the field teams will play to and/or which team gets first use of the ball, or similar questions in soccer matches, American football games, Australian rules football, volleyball, and other sports requiring such decisions.)* **Exercise 3)** What is the probability of getting a 'head' when tossing a coin? *There is no apparent reason for one side of a coin to land up any more often than the other – the coin is 'fair'. The probability of getting a 'head' when tossing a coin:* • *as a decimal: 0,5* • *as a fraction:* $\frac{1}{2}$ • *as a percentage: 50%.*	*lockstep* V: **coin flipping**

	Note: If you toss a coin you get either a head or a tail. *P(head) + P(tail)* = $\frac{1}{2} + \frac{1}{2} = 1$.	
Motivation (6 min)	**<u>Exercise 4)</u>** If we choose a letter at random from the word 'SUMS' what are possible outcomes? (*S,U,M*) what is the probability of obtaining: • the letter 'S'? ($^2/_4$) • the letter 'U'? ($^1/_4$) • the letter 'M'? ($^1/_4$) Sum up the probabilities of all possible outcomes: $P(S) + P(U) + P(M) = \frac{2}{4} + \frac{1}{4} + \frac{1}{4} = 1.$ Is there any pattern? The sum of the probabilities of all possible outcomes is 1.	*pair work* then one S writes his/her solution on the board
Motivation (3 min)	**<u>Exercise 5)</u>** How many possible outcomes are there? *Two possible outcomes – being late (L) and not being late (NL).* What is the sum of the probabilities of all possible outcomes? *The sum of their probabilities must add up to 1. Since* $P(L) + P(NL) = 1$, *it means that* $\frac{2}{9} + P(NL) = 1$. *So the probability of not being late is* $1 - \frac{2}{9} = \frac{7}{9}$. T: Let be 'I am late for work' an event *A*. Then 'I am not late for work' is its **complementary** event. Notation: *A´*.	*lockstep*
Presentation (5 min)	Let *A* be an event. *A´* is the event that occurs if *A* does not occur. *A´* is called a complementary event to A. *P(A´)*- **complementary probability**. **Rule:** Given the probability of an event, the probability of its complementary event can be found by subtracting the given probability from 1. So $P(A') = 1 - P(A)$. K javu *A* existuje jav *A'*, ktorý nastane vtedy, ak nenastane	V: **complement-ary probability**

	jav *A. Jav A´* - **doplnkový jav k javu *A*.**  Jav *A* - na kocke padne číslo menšie ako 3. Čo je potom *A´? Jav A'* - *na kocke padnú čísla 3,4,5,6*. *P(A')* je **doplnková pravdepodobnosť.** *P(A)+P(A')=1 a P(A')=1-P(A).*	
Practice (10 min)	 What is the complementary event of 'Petra has at least 20 years'? *Petra has at most 19 years.* **<u>Exercise 6)</u>** What is the complementary event of '*rolling a 5 or greater'?*Which of these events is more probable? Rolling a 5 or greater and rolling a 4 or less on a die are complementary events, because a roll is 5 or greater if and only if it is not 4 or less. The probability of rolling a 5 or greater is $\frac{2}{6} = \frac{1}{3}$, and the probability of rolling a 4 or less is $\frac{4}{6} = \frac{2}{3}$. Conclusion: it is more probable that Petra rolls a 4 or less than its complementary event. **<u>Exercise 7)</u>** *In theory, Kate should obtain a 6 on $^1/_6$ of her throws. Therefore, in theory she should throw a 6 on 5 of her 30 throws.* V: **high/ low probability**	*lockstep* *pair work* Ss try to estimate what is more probable then they count the accurate probability
Motivation (10 min)	**<u>Exercise 8 a)</u>** There are (at least) two ways how to solve this problem. **1)** One way is to think about all different possibilities:  How many different outcomes can the experiment have? *I could get a head on the first flip and a head on the second flip (H,H) or (H,T) or (T,H) or (T,T). So there are 4 equally likely outcomes.*  How many of those meet our conditions (they are favourable outcomes)? *Only one. So the probability of obtaining two tails is* $\frac{1}{4}$. **2)** There is another way you could think about this, and this	*lockstep*

	is because these events are *independent events*. It is very important idea to understand in probability. Some events are not independent, we will talk about them later. What happens in the first flip, in no way affects what happens in the second flip, and this is actually one thing that many people do not realize. There´s something called '**The Gambler´s Fallacy**' when someone thinks: 'If I got a bunch of heads in a row, then all of a sudden becomes more likely on the next flip to get tails.' That is not true. Every flip is independent event. If we make this assumption, we can say that the probability of getting tails and tails, or tails and then tails is the same thing as getting probability of getting tails on the first flip times the probability of getting tails on the second flip. So we obtain $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$, that is exactly what we got before. **<u>Exercise 8 b)</u>** *'A coin does not 'know' it came up heads before .... each <u>toss of a coin</u> is a perfect isolated thing.'* *If you use the assumption that these events are independent, we obtain:* $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$.	a T demonstrates both ways of solving the problem on the board V: **the Gambler´s Fallacy**
Presentation (4 min)	**Independent Events** How to find the probability of simultaneous occurrence of two or more independent events? *You can calculate the chances of two or more independent events by* ***multiplying*** *the chances.* A, B – independent events, then $P\ (A\ and\ B) = P(A) \cdot P(B) = P(A \cap B)$ Thus, the probability of simultaneous occurrence of two independent events is the product of their separate probabilities.	*S reads the definition*
Practice	**EXTRA Exercises**	*pair work*

(11 min)	**Exercise 9*** A die is rolled and a coin is tossed, find the probability that the die shows an odd number and the coin shows a head. **Solution**: Are these events dependent or independent? *Independent*. What is the probability that the die shows an odd number? $P(O)=\frac{3}{6}=\frac{1}{2}$. What is the probability that the coin shows a head? $P(H)=\frac{1}{2}$. What is the probability that the die shows an odd number **and** the coin shows a head? *Using the multiplication principle we obtain:* $P(O)\cdot P(H)=\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$. *So the answer is* $\frac{1}{4}$. **Exercise 10*** A nationwide survey found that 72% of people in Slovakia like pizza. If 3 people are selected at random, what is the probability that all three like pizza? **Solution** *Let L represent the event of randomly choosing a person who likes pizza.* $P(L)\cdot P(L)\cdot P(L) = 0.72\cdot 0.72\cdot 0.72 = 0.37 = 37\%$.	Extra exercises Ss work individually or in pairs, then Ss demonstrate their solution
Motivation (10 min)	**Combined events** - You already know that the probability of an outcome is: $P(A)=\frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$ However, finding the total number of possible outcomes is not always straightforward - especially when we have more than one event. Now we come across a problem that was proposed by Chevalier de Méré and is said be the start of famous correspondence between Pascal and Pierre de Fermat. They	T retells a story Ss are given a problem

	continued to exchange their thoughts on mathematical principles and problems through a series of letters. Historians think that the first letters written were associated with the above problem and other problems dealing with probability theory. Therefore, **Pascal** and **Fermat** are the mathematicians credited with the founding of probability theory. **<u>Exercise 11*</u>** Imagine that you were living in the seventeenth century as a nobleman. One day your friend Chevalier de Méré was visiting and challenged you to a game of chance. You agreed to play the game with him. He said, '*I can get a sum of 8 and a sum of 6 rolling two dice before you can get two sums of 7's.*' Would you continue to play the game?  How will we find out which sum is more probable? *We write down all the possibilities of obtaining 6,7 and 8 and so we will decide which outcome is more probable.* **Solution** 1^{st} die 1 2 3 4 5 6 2^{nd} die 1: 2 3 4 5 6 7 2: 3 4 5 6 7 8 3: 4 5 6 7 8 9 4: 5 6 7 8 9 10 5: 6 7 8 9 10 11 6: 7 8 9 10 11 12 T: I hope you changed your mind about playing the game of chance! (Chevalier de Méré did not state the order for the 8 and the 6.) • There are ten possible ways for Chevalier to get a favourable result. The probability of obtaining 6 or 8 is $\frac{10}{36}$.	during the lesson, they can get some bonus points if they solve it S writes on the board the sentence in italics *pair work* hint HW (unless Ss solve it during the lesson)

	• There are only six ways for you to get a favourable result. The probability of obtaining 7 is $\frac{6}{36}$. So it is more probable that you obtain 6 or 8. • Your chance of losing is $\frac{10}{16} = 0{,}625$. • I hope you imagined yourself as a wealthy nobleman, because you have a 62.5 percent chance of losing if you accepted to play the game with Chevalier de Méré.	
Conclusion (4 min)	HW – Solve all the problems on your worksheet. Watch: http://www.khanacademy.org/math/probability/v/independent-events-2 What is an **event**? *An **event** is one or more outcomes of an experiment (an event is the subset of all outcomes that can occur).* What is the **probability** of an event? ***Probability** is the measure of how likely an event is.* **What is the rule to determine the probability of the complementary event given the probability of an event?** ***Rule:** Given the probability of an event, the probability of its complementary event can be found by subtracting the given probability from 1. So P(A´) = 1 - P(A).* Which of these numbers cannot be a probability? **a) -0.00001** b) 0.5 **c) 1.001** d) 0 (nemožný jav) e) 1 (certain event) f) 20% **What are independent events?** *Two events A and B are said to be **independent**, if the occurrence or non-occurrence of one does not affect the probability of the occurrence (and hence non-occurrence) of the other.* T: Thank you for your attention. Have a nice day!	*lockstep*

Appendix B

Worksheet 7 – **Introduction to PROBABILITY**

'When faced with two choices, simply toss a coin. It works not because it settles the question for you, but because in that brief moment when the coin is in the air, you suddenly know what you are hoping for.' Anonymous

Fill the missing words and numbers.

An **event** is one or more outcomes of an experiment (an event is the subset of all outcomes that can occur).

Ex. 1) Throw a fair die (plural: dice).

a) What is the probability of the event 'obtaining 6'?

The probability of 'obtaining 6' when throwing a die:
as a decimal: _____ *, as a fraction:* _____ *, as a percentage:* _____ .

b) What is the probability of 'obtaining an even number'?

All the possible outcomes are: _____ *. The number of possible outcomes:* _____ .
All the favourable outcomes are: _____ *. The number of favourable outcomes:* _____ .
The probability of 'obtaining an even number' is: _____ .

LANGUAGE

the probability of (doing) (SYN likelihood, chance)
probable (likely to happen), **probably**
event
equally likely
more likely
favourable(BrE) **/favorable** (AmE)
possible
coin flipping

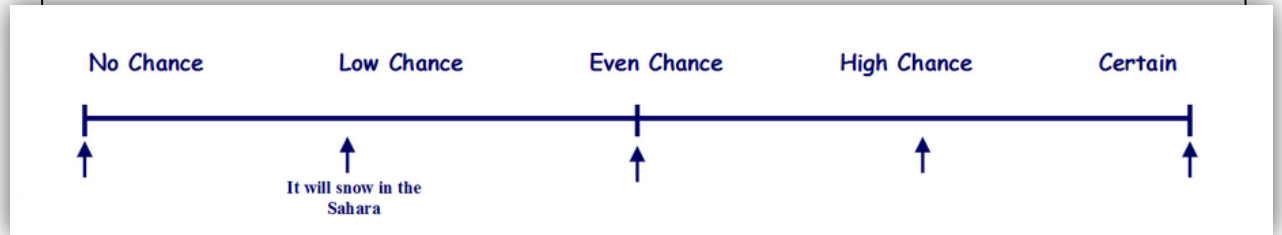
Probability is the measure of how likely an event is. Notation for the probability of the event *A* is *P(A)*.

$$P(A) = \frac{\text{number of } \_\_\_\_ \text{ outcomes}}{\text{number of } \_\_\_\_ \text{ outcomes}}$$

Pravdepodobnosť náhodných udalostí definujeme ako pomer počtu všetkých priaznivých výsledkov a všetkých možných výsledkov.
Let A be an event that is

- *certain* to occur. Then $P(A) =$ _____ .
- *impossible* to occur. Then $P(A) =$ _____ .
- *possible* to occur. Then _____ $\leq P(A) \leq$ _____ .

Ex. 2) Put arbitrary events on the probability scale. One has been done for you.



Ex. 3) What is the probability of getting a 'head' when tossing a coin?

Ex. 4) If we choose a letter at random from the word 'SUMS', what is the probability of obtaining the letter 'S'? The letter 'U'? The letter 'M'? What is the sum of the probabilities of all possible outcomes?

The **sum of the probabilities of all possible outcomes is** _____.

Ex. 5) The probability that I am late for work tomorrow is $^2/_9$. What is the probability that I am not late for work?

Let *A* be an event. *A´* is the event that occurs if *A* does not occur. *A´* is called a **complementary event** to *A*. *P(A´)* – **complementary probability.**

Rule: Given the probability of an event, the probability of its complementary event can be found by subtracting the given probability from 1. So *P(A´)* = _____.

Ex. 6) What is the complementary event of '*rolling a 5 or greater'?* Which of these events is more probable?

Ex. 7) Kate and Josh each throw a die 30 times. How many times would you expect Kate to obtain a 6?

Two events A and B are said to be **independent**, if the occurrence or non-occurrence of one does not affect the probability of the occurrence (and hence non-occurrence) of the other. If A and B are independent events, then

P (A and B)= _____ = *P(A ∩ B).*

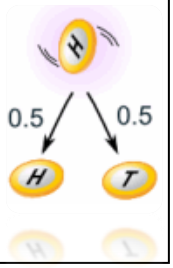
Ex. 8)

a) We flip a coin twice. What is the probability of getting two tails?

b) We toss a coin three times. What is the probability of getting heads, then tails, then heads?

A coin does not 'know' it came up heads before ...

.... each flip of a coin is a perfect isolated thing.



Appendix C

Lesson plan 8 – **Pascal Triangle and Binomial Theorem**

Subject: Mathematics **Time**: 90 min

Theme: Pascal Triangle and Binomial Theorem **Grade**: 3rd graders

Content objectives	Students will be able to • state the basic patterns of the Pascal triangle; • evaluate $(\boldsymbol{a}+\boldsymbol{b})^{\boldsymbol{n}}, \boldsymbol{n} \in \boldsymbol{N}$ using the binomial theorem.
Language objectives	Students will be able to • read polynomials; • apply rules to the plurals of numerical terms such as *dozen*, *score*, *hundred*, *thousand*, *million*, and similar.
Formative objectives	Students will be able to • cooperate during pair/group work. • tolerate each other during pair/group work.

Language for learning: equilateral triangle, Pascal triangle, arbitrary, binomial expansion

Materials: worksheets

Stage & Time	Procedure	Notes
Introduction Warm up (3 min)	T: It's that time of the week again. Time to tickle your brain. The brain is the centre of the nervous system in the human body. Did you know that the brain knows everything but it can't feel a thing? (there are no pain receptors in the brain). **Brain Teaser:** Farmer Giles has four sheep. One day, he notices that they are standing in such a way that they are all the same distance away from each other. That is to say, the distance between any two of the four sheep is the same. How can this be so? **Solution**: The sheep are standing on the four corner points	(biology) - cross curriculum issue *group work*

	of an equal-sided pyramid (a tetrahedron). Or to put it another way, three are on the points of an equilateral triangle and the other is on a mound of earth in the centre.	V: **equilateral triangle**
Revision (2 min)	Revision of properties of binomial coefficient. Ss evaluate the following: a) $\binom{n}{0} = 1$; b) $\binom{n}{k} = \binom{n}{n-k}$ (symmetry); c) $\binom{n}{1} = n$; d) $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$.	*lockstep*
Motivation + Presentation (10 min)	One of the most interesting number patterns is Pascal's Triangle. Who was Blaise Pascal? Are you familiar with the Pascal Law? What do you know about the Pascal Triangle? ***Blaise Pascal*** *was a French mathematician, physicist, inventor, writer and theologian. Pascal developed the probability theory. Originally applied to gambling, today it is extremely important in economics.* **PASCAL'S TRIANGLE** **Pacsal´s Triangle** **Evaluate** **Row Sum** $\binom{0}{0}$ 1 2^0 $\binom{1}{0}$ $\binom{1}{1}$ 1 1 2^1 $\binom{2}{0}$ $\binom{2}{1}$ $\binom{2}{2}$ 1 2 1 2^2 $\binom{3}{0}$ $\binom{3}{1}$ $\binom{3}{2}$ $\binom{3}{3}$ 1 3 3 1 2^3 ... ... $\binom{n}{0}$ $\binom{n}{1}$ $\binom{n}{2}$ ... $\binom{n}{n-2}$ 2^n	**Pascal Triangle –** Pascalov trojuholník T writes Pascal´s Triangle on the board

	Pascal's Triangle was originally developed by the ancient Chinese, but Blaise Pascal was the first person to discover the importance of the patterns it contained. Look at your diagram. What patterns can you see? *Each number is the total of the two numbers above it.* Write this pattern using two <u>arbitrary</u> binomial coefficients. Using this property, evaluate: $\binom{n}{k} + \binom{n}{k+1}$. Answer: $\binom{n+1}{k+1}$. *We have already mentioned this property of binomial coefficients.* What do you notice about the **horizontal sums** (row sums)? Is there a pattern? Can you predict the next total? *It doubles each time (powers of 2).* And the triangle is also <u>symmetrical</u> (the numbers on the left side have identical matching numbers on the right side, like a mirror image). What property of the binomial coefficients secures this property? *It is caused by the equality:* $\binom{n}{k} = \binom{n}{n-k}$. What is the opposite of the adjective **symmetrical**? (*asymmetrical)*	V: **arbitrary** V: **symmetry/ asymmetry**
Presentation (8 min)	**The Binomial Theorem** Write down formulas $(a+b)^n$, for $n = 1, 2, 3$ and compare them with the Pascal Triangle. $$(\boldsymbol{a+b})^{\boldsymbol{1}} = \boldsymbol{a+b}$$ $$(\boldsymbol{a+b})^{\boldsymbol{2}} = \boldsymbol{a^2 + 2ab + b^2}$$ $$(\boldsymbol{a+b})^{\boldsymbol{3}} = \boldsymbol{a^3 + 3a^2b + 3ab^2 + b^3}$$ *It is clear that the coefficients in front of the variables correspond to the particular rows of the Pascal triangle.* The **binomial theorem** describes the algebraic expansion of powers of a binomial.	3 Ss at the board V: **the binomial expansion** (binomický

	The Binomial Theorem: $(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \cdots +$ $+\binom{n}{n-1}a^1 b^{n-1} + \binom{n}{n}a^0 b^n\ ,\quad n \in N.$ We will not prove the theorem. How many terms appear in the binomial expansion for $(\boldsymbol{a+b})^{\boldsymbol{n}}$? *(n+1 terms)*	rozvoj)
Practice (15 min)	**Exercise 1** Using the binomial theorem evaluate **a)** $(\boldsymbol{2a+b})^{\boldsymbol{6}}$ **Solution**: Using the Binomial Theorem we obtain $\binom{6}{0}(2a)^6 b^0 + \binom{6}{1}(2a)^5 b^1 + \binom{6}{2}(2a)^4 b^2 + \binom{6}{3}(2a)^3 b^3$ $+\binom{6}{4}(2a)^2 b^4 + \binom{6}{5}(2a)^1 b^5 + \binom{6}{6}(2a)^0 b^6 =$ $= 2^6 a^6 + 6.2^5 a^5 b^1 + 15.2^4 a^4 b^2 + 20.2^3 a^3 b^3$ $+15.2^2 a^2 b^4 + 6.2a^1 b^5 + b^6.$ $\boldsymbol{b)}\ \boldsymbol{2^n} = (1+1)^n = \binom{\boldsymbol{n}}{\boldsymbol{0}} + \binom{\boldsymbol{n}}{\boldsymbol{1}} + \binom{\boldsymbol{n}}{\boldsymbol{2}} + \cdots + \binom{\boldsymbol{n}}{\boldsymbol{n-1}} + \binom{\boldsymbol{n}}{\boldsymbol{n}}.$ It is the sum of the *n*-th row of Pascal triangle. In other words, the sum of the entries in the *n*th row of Pascal's triangle is the *n*-th power of 2. (Ss may have noticed this property earlier.) * MORE about the Pascal Triangle: What is the definition the binomial coefficients? *If the set has n elements, the number of k-combinations is equal to the binomial coefficient.* So what does the number $\binom{n}{0}$ express? *It is the number of zero-element subsets of an n- element set.* So what does the number $\binom{n}{1}$ express? *It is the number of one-element subsets of an n- element set.* If we sum up $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n-1} + \binom{n}{n}$ we will obtain the number of all subsets of a set containing	1 S at the board *rozširujúce učivo *lockstep*

	n elements. **How many subsets (of any size) are there for a set of five elements?** *It is the sum of 5-th row of Pascal triangle. Since we know that* 2^n *is the sum of the n-th row of Pascal triangle, there are* $2^5 = 32$ *subsets for a set of five elements.* ***c***) $\left(\mathbf{3x^2 - \frac{y}{2}}\right)^{\mathbf{4}} = 81\,x^8 - 54x^6y + \frac{27}{2}x^4y^2 - \frac{3}{2}x^2y^3 + \frac{y^4}{16}$. ***d***) the term of the binomial expansion of $\left(\frac{\mathbf{2}}{\boldsymbol{x^3}} + \mathbf{3x^2}\right)^{\mathbf{5}}$ which does not contain x. *The fourth term. Its value is 1080.* How do we call the term which <u>does not contain</u> *x* in Slovak language? *Absolútny člen.* **e)** that term of the binomial expansion of $\left(x + \frac{1}{x}\right)^8$ which contains x^4. *The third term:* $\binom{8}{2}x^6.\frac{1}{x^2} = 28x^4$. **Exercise** Let $n \in N$. Which of the following binomial coefficients is the largest? $$\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \ldots, \binom{n}{n-1}, \binom{n}{n}.$$ **Solution** The solution depends on *n*, if it is odd or even. If *n* is even, then the answer is: $\binom{\boldsymbol{n}}{\boldsymbol{n/2}}$. If *n* is odd, then the answer is $\binom{\boldsymbol{n}}{\boldsymbol{(n-1)/2}}$.	
Revision (5 min)	• $3^2 = 9$ – 'three squared equals nine' or 'the square of three equals nine'; • $2^4 = 16$ – 'two to the power of four equals sixteen' or 'the fourth power of two equals sixteen'; • $\sqrt[5]{32} = 2$ – 'the fifth root of thirty-two equals two'. **<u>NUMBERS</u>** **Hundreds** and **tens** are usually separated by 'and' (in American English 'and' is not necessary). 110 – one hundred *and* ten	*lockstep* Language for learning

	1250 – one thousand, two hundred *and* fifty 2001 – two thousand *and* one  Read aloud: 928 287. **Solution**: • nine hundred and twenty-eight thousand and two hundred and eighty-seven; • nine hundred twenty-eight thousand, two hundred eighty-seven Rules to the **Plurals of numerical terms** such as *dozen, score, hundred, thousand, million,* and similar.  When do we use plural 's' after *hundred, thousand, million,...?* The following rules apply to the plurals of numerical terms such as *dozen, score, hundred, thousand, million,* and similar. • When these numerical terms are modified by another number or expression such as *one, three, a few / several, etc.* they take SINGULAR form. It is when they are a part of a number, e.g. *two million, four score.* • When these terms are on their own, they can be PLURAL, e.g. *millions of insects, hundreds of Euros, thousands of light years.*	
Practice (2 min)	**dozen**: I bought **three** _____mangoes. (*dozen)* She has _____ **of handbags. (dozens)** **hundred**: I bought this camera for **two** _____dollars. *(hundred)*	
Conclusion 2 min	 When do we use plural 's' after *hundred, thousand, million,...?*  When is Pascal' s triangle helpful?  What does the binomial theorem describe?	*lockstep*

	HW: USE mathematical notation for these expressions: a) the sum of the second powers of three consecutive natural numbers. **Solution**: For example: $n^2 + (n+1)^2 + (n+2)^2,\ n \in N$; b) the fifth root of the product of the squares of two consecutive natural numbers. **Solution**: For example: $\sqrt[5]{n^2 \cdot (n+1)^2}$ **Non-compulsory HW:** Find English jokes based on English puns concerning school. Example: Math teachers have lots of problems.	

Appendix D

Worksheet 8 – **Pascal Triangle and the Binomial Theorem**

'The supreme function of reason is to show man that some things are beyond reason.' Blaise Pascal

Brain Teaser: Farmer Giles has four sheep. One day, he notices that they are standing in such a way that they are all the same distance away from each other (the distance between any two of the four sheep is the same). How can this be so?_____

_____

Write at least three contributions of Blaise Pascal. _____

_____

_____

Pascal's Triangle **Evaluate** **Row Sum**

$\binom{0}{0}$ ____

$\binom{1}{0}\ \binom{1}{1}$ ____

$\binom{2}{0}\ \binom{2}{1}\ \binom{2}{2}$ ____

$\binom{3}{0}\ \binom{3}{1}\ \binom{3}{2}\ \vdots\ \binom{3}{3}$ ____

$\vdots$ ...

$\binom{n}{0}\ \binom{n}{1}\ \binom{n}{2}\ \dots\ \binom{n}{n-2}\ \binom{n}{n-1}\ \binom{n}{n}$ ____

The Binomial Theorem: $n \in N;\quad a, b \in R;$

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}a^1 b^{n-1} + \binom{n}{n}a^0 b^n.$$

1) Using the binomial theorem evaluate

a) $(2a+b)^6 =$

b) $2^n = (1+1)^n =$

c) $\left(3x^2 - \frac{y}{2}\right)^4 =$

a) the term of the binomial expansion of $\left(\frac{2}{x^3}+3x^2\right)^5$ which does not contain x;

b) the term of the binomial expansion of $\left(x+\frac{1}{x}\right)^8$ which contains x^4.

2) Let $n \in N$. Which of the following binomial coefficients is the largest? Why?
$\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \ldots, \binom{n}{n-1}, \binom{n}{n}$.

Language for Learning:

Singular or plural of numerical terms such as *dozen*, *score*, *hundred*, *thousand*, *million*.

3) Circle the correct word and add more examples.

- When these numerical terms are modified by another number or expression such as *one, three, a few / several, etc.* they take SINGULAR/ PLURAL form. It is when they are a part of a number.
  e.g. *two million*, *four score*, _____.
- When these terms are on their own, they can be SINGULAR/ PLURAL.
  e.g. *millions of insects, hundreds of Euros, thousands of light years,* _____.

4) FILL the missing word.

dozen: I bought **three** _____mangoes. She has _____ **of handbags.**

hundred: I bought this camera for **two** _____dollars.

5) USE mathematical notation for these expressions:

a) The sum of the second powers of three consecutive natural numbers.

b) The fifth root of the product of the squares of two consecutive natural numbers.

Appendix E

Test Combinatorics

Test consists of 9 tasks. 8 of them are obligatory; the last one is the bonus task focusing on mathematics. The first 5 exercises focus mainly on mathematics. The last 3 exercises focus on English language. Describe your solutions in English. Do your best! (Note that p = point)

1) Consider the set {*K, L, M*}. Write down at least **three** examples of

   a) two-element combinations without repetition of this set (1 p),

   b) variations of 2 distinct elements of this set (1 p).

2) Match (one number has no match) (2 p)

a) The number of *k*-combinations with repetition from a set containing *n* elements is given by	$\frac{\boldsymbol{n}!}{\boldsymbol{n_1}!\boldsymbol{n_2}!\ldots\boldsymbol{n_k}!}$
b) If a set has *n* elements, then **a variation without repetition** is the ordering of *k* objects if any object cannot be chosen more than once. The number of variations is	$\frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{k})!}$
	$\binom{\boldsymbol{n}+\boldsymbol{k}-\boldsymbol{1}}{\boldsymbol{k}}$

3) A coin is tossed, a die is rolled, and a card is drawn from a pack containing 32 cards. How many possible outcomes does this experiment have? (3 p)

4) From twenty tickets in a hat, four tickets are to be chosen at once. In how many different ways can the four tickets be chosen? Evaluate. (3 p)

5) How many distinct arrangements can be formed from all the letters of MISSISSIPPI? (3 p)

6) Fill the missing expressions. (4 p)

We write this expressions:	We 'read' this expressions in English:
$\frac{4!}{5!}$	
	a set containing four elements *A,B,C,D*
$\binom{4}{3}$	
8 x 3 = 24	

7) Circle the correct form (more than one possibility can be correct) (3 p):

'Almost each couple go / goes on holiday to the mountains.'

Give **two** more examples of such (collective) nouns:

8) Circle the correct answer (more than one possibility can be correct) (3 p):

'He is not to stay here for the weekend.' It means that

a) it is the weekend now and he is not here.
b) he is not allowed to stay here during the weekend.
c) he was not with us last weekend.

Produce your own sentence using expression BE TO.

BONUS

Four employees at a company picnic are to stand in a row for a group photograph. In how many ways can this be done if Jane and John want to stand next to each other? (2 p)

Appendix F

Univerzita Mateja Bela v Banskej Bystrici, Fakulta prírodných vied

Dotazník postojov študentov k výučbe s využitím metódy CLIL

Zúčastnili ste sa výučby matematiky metódou CLIL – Content and Language Integrated Learning. Vo Vašom prípade bol obsah predmetu matematiky sprostredkovaný v anglickom jazyku. Tento anonymný dotazník sa zaoberá Vaším postojom k výučbe matematiky touto metódou. Je to súčasť mojej diplomovej práce a vyplnením tohto dotazníka mi pomôžete.

Za spoluprácu počas hodín a za čas, ktorý venujete vyplneniu dotazníka Vám ďakujem.

Alžbeta Brišová, 2012.

1) Myslíte si, že matematika je vhodný predmet na integrovanie?

   áno nie neviem

2) Uvítali by ste výučbu touto metódou aj v iných predmetoch?

   áno nie neviem

3) Myslíte si, že sa s matematickou terminológiou v angličtine stretnete aj v budúcnosti?

   áno nie

4) Výučba pomocou tejto metódy bola pre mňa

   veľmi náročná skôr náročná primerane náročná nenáročná

5) Jednoduchší pre mňa bol

   obsah (matematika) jazyk (anglický)

6) Myslím, že keby sa učilo tradične, tak by som si kombinatorické poznatky osvojil/a

   viac porovnateľne menej

   ako keď bola táto téma vyučovaná metódou CLIL.

7) Môj postoj k metóde CLIL je

   pozitívny neutrálny negatívny

8) <u>Pozitíva</u> tejto výučby vidím v:

   <u>Negatíva</u> tejto výučby vidím v:

9) Kvalita mojich jazykových zručností sa zlepšila:

určite áno asi áno asi nie určite nie

Ak bola Vaša odpoveď určite/asi áno, tak zakrúžkujte, ktoré jazykové zručnosti ste si zlepšili:

a) porozumenie ústnym prejavom,
b) porozumenie písomným prejavom,
c) schopnosť prijímať informácie,
d) schopnosť interpretovať a odovzdávať informácie,
e) schopnosť dedukcie a analýzy písaného/hovoreného textu,
f) schopnosť vyjadriť a zdôvodniť svoj názor,
g) plynulosť môjho hovoreného prejavu,
h) presnosť vo vyjadrovaní.

10) Aká bola slovná zásoba?

ťažko dedukovateľná ľahko dedukovateľná nezrozumiteľná

11) Usporiadajte podľa dôležitosti, čo považujete pre seba za najviac/najmenej prínosné pri výučbe touto metódou (1- najviac prínosné, 8 - najmenej prínosné)

sebavedomie
motivácia
jazykové zručnosti
spolupráca s inými
matematický obsah
interkultúrna komunikácia
aktívna komunikácia
slovná zásoba

12) Celkovo hodnotím hodiny na škále 1 – 5 číslom (1- najlepšie, 5- najhoršie):

13) Aký je váš názor na túto metódu? Zhodnoťte niekoľkými slovami.

ĎAKUJEM za čas aj úprimné odpovede. ☺